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Forecasting Short-Term Electricity Load Profiles

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Forecasting Short-Term Electricity Load Profiles

A Joint AEPCO - University of Arizona Project

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Executive Summary

AEPCO and the University of Arizona's Department of Agriculture and Resource Economics (AREC) collaborated during the fall semester 2005 on a project to improve forecasts of next-day electricity load. The project was conducted as part of an AREC M.S. class in applied econometrics. Students developed econometric models for forecasting next-day hourly load profiles, and delivered results to AEPCO in a formal business presentation in December 2005.

The particular econometric models developed are known as ARIMA (autoregressive, integrated, moving average) models which use only past load data to forecast next-day load profiles. The models were calibrated for five distinct seasons in 2004: winter, spring, premonsoon, monsoon, and fall periods. The ARIMA models were estimated using rolling samples of 28 days of (672) hourly load observations for one week in the five seasons. ARIMA forecasts yielded reasonable results: forecast errors at coincidental peaks were generally at or below 5 percent. The time of day of coincidental peaks was usually forecast correctly. ARIMA forecast also captured the shape of 24-hour profiles adequately.

The UofA ARIMA forecasts were compared to AEPCO pre-planning forecasts for two weeks in the summer of 2005—a pre-monsoon week in June and a monsoon week in August. ARIMA models usually provided modest improvements relative to AEPCO forecasts. Mean absolute percentage errors (MAPE) for the ARIMA forecasts tended to be slightly smaller than the MAPE of AEPCO's forecasts.

ARIMA forecasts can be updated daily by AEPCO econometricians with less than ¹/₂ hour of work. In the short term, AEPCO staff can use ARIMA forecasts to complement their pre-planning forecasts. ARIMA and pre-planning forecasts can be compared systematically to yield further improvements in forecasting next-day load profiles.

Acknowledgments

Several individuals at AEPCO and the University of Arizona played key roles in facilitating this project. Dennis Criswell and Dirk Minson of AEPCO as well as Alan Ker of the Department of Agricultural and Resource Economics were involved in initial conversations about the possibility of such a joint project. Without their vision and willingness to undertake a new kind of joint venture between a cooperative and a public university, this project could not have been completed. Clifford Cathers and Mike Newton of AEPCO facilitated the initial meeting between AEPCO and University of Arizona participants, and continued to meet throughout the semester with UofA participants. In October 2005, Walter Bray and his staff provided invaluable information about how next-day forecasting to AEPCO is conducted. Finally, the students involved in the project—David Brettler, Katie Pittenger, and Ted Stockton—displayed admirable maturity and discipline in conducting research on forecasting, and in delivering a useful formal presentation to AEPCO on December 2, 2005.

Gary Thompson

Professor Agricultural & Resource Economics January 2006

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Forecasting Short-Term Electricity Load Profiles

AEPCO and the University of Arizona's Department of Agriculture and Resource Economics (AREC) agreed in June 2005 to explore opportunities for collaboration in forecasting next day hourly load. Mr. Clifford Cathers of AEPCO developed a detailed proposal outlining specific objectives for improving forecast accuracy. Dr. Gary Thompson of the University of Arizona ("UofA"), AREC, agreed to coordinate the department's efforts and conduct the project in connection with his graduate course, *Advance Applied Econometrics*. Three graduate students in the class, assisted by Dr. Thompson, developed several time series models for accurately forecasting short-term hourly loads.

The objectives of the project were to:

- 1. Forecast next-day coincidental peak loads for Class A members
- 2. Forecast next-day 24-hour load profile for Class A members
- 3. Forecast next-day 24-hour load profile for each Class A member
- 4. Forecast next-day 24-hour load profile by delivery point and substation

The objectives were designed for a project to be conducted over the course of several years. The first year's installment was to forecast coincidental peak load for Class A members. Next, UofA was to develop a 24-hour load profile for Class A Members. In subsequent years, the project would expand to forecast 24-hour load profiles for each Class A member. The final step will be to provide 24-hour load profile for each delivery point and substation.

Although the initial idea was to focus solely on the first objective, UofA econometricians realized the first two objectives could be tackled simultaneously. Accordingly, results presented here include analyses of both coincidental peak load forecasting performance and 24-hour load profile forecasting performance.

Business Problem

For building useful models, econometricians need to first understand the underlying economic and business problems which they want to model. In the current context, UofA econometricians sought to formulate a concise statement and explanation of the business problem underlying the need for improved short-term forecasts. Without an understanding of the business problem, UofA econometricians were afraid their forecasting results might not be relevant for improving daily decision making at AEPCO.

UofA econometricians think AEPCO's short-run business problem is one of costminimization as opposed to profit maximization. Regulatory strictures on electricity rates preclude AEPCO from passing on increases in input costs to its members by raising electricity prices. Hence, revenues in the short run are only affected by the amount of electricity demanded at the Class A level because electricity prices are fixed. Consequently, the benefit of improved accuracy of AEPCO's pre-planning forecasts is manifested in the ability to lower input costs for generating electric power. Also, to the extent surplus power is sold on the spot market or shortfalls in generation must be met by purchases on the spot market, more accurate forecasts can minimize reliance on potentially volatile day- or hour-ahead spot-markets.

A flow chart of the AEPCO's short-run business problem is depicted in figure 1. The first two objectives mentioned above relate most closely to this business problem of least-cost electricity generation and buying or selling electricity in the spot market.



Figure 1. Short-Run Business Problem

<u>Costs of Generating Electricity</u>. Accurate pre-planning forecasts of coincidental peak loads and 24-hour load profiles may allow AEPCO to adjust the optimal daily mix of coal vs. natural gas for generating electricity at its Apache Junction plant. The least-cost mix of coal and natural gas is essential in determining optimal capacity utilization and operating efficiency. Natural gas is becoming more expensive (see Figure 2) and AEPCO must contract a day in advance for adequate delivery of natural gas. More accurate forecasts will allow AEPCO to contract for precisely the correct amount of expensive natural gas to burn only when necessary.



Figure 2. U.S. Prices of Natural Gas

Source: U.S. Department of Energy, http://tonto.eia.doe.gov/dnav/ng/hist/n9190us3m.htm

<u>Spot Market Transactions</u>. Improved forecasts may allow higher returns on transactions in the day- and hour-ahead spot markets for electricity should the spot market revitalize. Although volatility in the day- and hour-ahead spot markets is not currently a problem, if generating capacity in the industry lags demand, a more active, volatile spot market may re-emerge. In such a volatile market, more accurate short-term forecasts could be an essential tool for minimizing exposure to volatile spot market prices.

<u>Costs of Maintaining Transmission Networks</u>. Underlying these explicit costs of generating electricity and selling or buying on the spot market is AEPCO's ongoing commitment to service and maintenance of its transmission network. More detailed forecasts at the substation and delivery point can improve scheduling decisions and minimize disruptions in service. Objectives 3 and 4 above relate to this slightly longer run business problem of servicing and maintaining transmission lines, which are an integral part of delivering generated power to Class A members.

Impacts of Market Forces

Market forces may continue to enhance the value of accurate short-term forecasts. Demand for electricity at the Class A member level has increased substantially over the past decade. Peak demands have grown at an even faster pace than average demands over the same period (see Figure 3). As a result, small percentage errors in forecasts become more costly when applied to growing peak loads.



Figure 3: Monthly Average and Maximum Loads, AEPCO 6 Class A Members

Source: Calculations from AEPCO Load Data.

Other market forces are likely to make generation of electricity more costly. Natural gas prices have increased in real terms since 2001. And relative to the price of coal, natural gas has become much more expensive. Hence, errors in forecasting become more costly when peak demands must be met by burning increasingly more expensive natural gas. If El Paso Corporation, AEPCO's provider of natural gas, is successful in proposing so-called hourly burn fines, the costs of forecast errors could become even higher. If natural gas prices increase, if spot markets become more volatile, or if load levels continue to grow and magnify the costs of small percentage errors in forecasts, statistically derived forecasts may prove even more beneficial to AEPCO in making short-term decisions aimed at maximizing returns for their member owners.

AEPCO's Pre-Planning Forecasts

AEPCO's pre-planning department currently relies on expert judgment whereby intuition, experience, and practical expertise are combined to generate pre-planning forecasts of next-day load profiles. AEPCO forecasters take into account recent load shape, current forecasted weather, day of the week, holidays, and trends in formulating their forecasts. AEPCO forecasters typically use the previous one or two months of load data in formulating their forecasts. With these data, AEPCO forecasters review recent load history for the day and hour most similar to the current forecasted period. Analysts then adjust the anticipated load level to account for observable factors—day of the week, holidays, etc.— likely to affect the current forecasted load level compared to the most similar recent historical observation.

Monday through Thursday, pre-planners forecast next-day hourly loads. On Friday, loads for the following three days are forecast, resulting in a continuous series of 72 hourly loads through midnight of the following Monday. All loads are forecast for all Class A members jointly; no member-by-member forecasts are generated daily.

Alternative Forecasting Methods

In the process of considering alternative methods for forecasting, AEPCO personnel suggested a successful alternative forecasting model should have several characteristics. First, an alternative model should be able to generate a forecast within 15 to 30 minutes each day. Second, the model should accurately forecast load levels while capturing the shape of the daily load profile. Third, the model should account for weather effects, such a cumulative heating degrees and highly variable monsoon conditions. Fourth, the model should adapt to the changing shapes of daily load profiles. While winter and summer daily profiles typically follow regular patterns, daily profiles in transitional periods in the spring and fall can vary in shape from one day to the next. Any statistical model worth adopting must be sufficiently flexible to capture these changing daily profiles.

AEPCO personnel estimate their pre-planning forecasts yield forecast errors in the vicinity of about 5 percent. Existing methods using expert judgment appear to have been sufficiently accurate for AEPCO's current load levels. But the option of using other statistical methods should not be viewed as an all-or-nothing adoption decision. AEPCO can continue to employ expert judgment methods while comparing their daily forecasts to those derived from statistical models. Comparisons of the two types of forecasts could offer AEPCO forecasters new insights into their expert judgment. Conversely, econometricians at AEPCO may refine their statistical forecasting models based on insight from expert forecasters. Both sets of forecasts could also be evaluated after the fact using various metrics so that strengths and weaknesses of each could be identified.

Statistical Models for Short-Term Forecasting

Statistical models for forecasting short-term electrical loads fall into three broad categories:

- 1. Time Series Models
- 2. Artificial Neural Networks
- 3. Semi- and Non-Parametric Regression

Each of the class of models has desirable features and each, in turn, has weaknesses. A detailed discussion of the pros and cons of implementing and estimating models in each class is not included here. What is ultimately of concern to forecasters is simply forecast error. Based on most commonly adopted metrics of forecast error, none of the models in any of the three classes of models performs measurably better in all circumstances. In various literature reviews, each of the types of models are capable of producing mean absolute percentage errors of as low as 2 percent (EPRI). From the perspective of minimizing forecast errors, no class of models is clearly preferred to another. Forecast performance of particular models depends on the specific load and other data used, making comparisons across models difficult.

ARIMA Models

For present purposes, one type of time series model was selected for forecasting: ARIMA. ARIMA is an acronym standing for AutoRegressive, Integrated, Moving Average models. In its simplest form, an ARIMA model postulates current load is simply some function of past loads. For that reason, it is referred to as a univariate model because the only variable entering the model is load. The only data required to estimate the ARIMA model are time series observations on loads. But ARIMA models can be augmented with other variables. If the other variables are non-stochastic such as dummy variables for day of the week or holidays, ARIMA models using these non-stochastic variables are used for *intervention analysis*. If stochastic variables such as actual and predicted weather variables are used, they are included in the ARIMA model by means of a *transfer function*.

In what follows, some slightly more technical explanation of ARIMA models is provided. Readers not interested in the technical details should skip to the section titled Seasonal Forecasting on page 20.

An ARIMA model is specified generally as ARIMA(p,d,q) where p denotes the order of the autoregressive (AR) component, d denotes the order of differencing of the raw data, and qindicates the order of the moving average (MA) process. It is convenient to introduce concise notation as a means to make more explicit the autoregressive and moving average portions of the model. A typical *p*-order autoregressive model of a sequence $\{y_t\}$ of load can be written as

$$\left(1-a_1L^1-a_2L^2-\cdots-a_pL^p\right)y_t=a_o+\varepsilon_t$$

in which *L* denotes the lag operator, i.e. $L^i y_t \equiv y_{t-i}$. Parameters to be estimated are given by the a_i 's and ε_t is the white noise error. The polynomial in the lag operators can be depicted even more compactly as

$$A(L)y_t = a_o + \varepsilon_t$$

where A(L) is shorthand for the polynomial. Similarly, the moving-average portion of the ARIMA can be depicted as

$$y_t = a_o + a_1 y_{t-1} + a_2 y_{t-2} + \dots + a_p y_{t-p} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \dots + \beta_q \varepsilon_{t-q}$$
$$A(L) y_t = a_o + B(L) \varepsilon_t$$

Here, the polynomial in the moving average is denoted as B(L).

Because many time series display marked trends, the univariate time series is often "filtered" in some manner so as to remove any detectable trends. First differences,

 $\Delta y_t \equiv y_t - y_{t-1}$, and second differences, $\Delta^2 y_t = \Delta (y_t - y_{t-1}) = y_t - 2y_{t-1} + y_{t-2}$, are usually sufficient to remove linear trends. To remove nonlinear trends, the raw series is first transformed by the natural logarithm and then differenced. If no filtering is performed, the model is simply an ARMA(*p*,*q*) model.

The preceding ARIMA(p,d,q) model is *additive* in all parameters. For purposes of modeling seasonal patterns, *multiplicative* ARIMA models are often specified. The general notation for a seasonal multiplicative model is

$$\operatorname{ARIMA}(p,d,q)(P,D,Q)_{s}$$

where

p = the order of the non-seasonal AR process

q = the order of the non-seasonal MA process

d = the number of non-seasonal differences

P = the number of *multiplicative* autoregressive coefficients

D = the number of seasonal differences

Q = the number of *multiplicative* moving average coefficients

s = the seasonal period

For application to forecasting hourly load demands, "seasonality" refers to the period over which one cycle of a pattern occurs such as a day (24 hours) or a week (168 hours). For examples of multiplicative seasonal ARIMA models, see Enders, pp. 93-99.

Box and Jenkins formalized analysis of time series by using ARIMA models. They outline three distinct steps in their analysis: identification; estimation; and forecasting. Each step will be addressed in turn.

Identification. Identification consists in choosing parameters of the ARIMA model so that the remaining error is white noise or, possibly, Gaussian white noise. In linear models, identification is often achieved by zero restrictions, which exclude certain parameters from the model, and

normalizations, which typically set parameters equal to one. In ARIMA models, finding the appropriate order of the AR and MA portions of the models amounts to imposing zero restrictions on a more general model. In intuitive terms, the analyst seeks to include only the necessary parameters in the ARIMA model such that all trends and systematic patterns can be captured by the model. The only remaining components in the model should be purely random and, therefore, inherently unpredictable.

Identification is usually aided by various graphical methods and statistical tests are conducted to verify whether error terms behave as white noise should. Using graphical methods, one inspects the autocorrelation and partial autocorrelation functions to ascertain the appropriate autoregressive (AR) and moving-average (MA) specifications.

Prior to ascertaining the order of the AR and MA specifications, the data are filtered. In an effort to mimic pre-planning forecast techniques, samples of 28 days (672 hourly observations) are employed. The datasets are *rolling* samples inasmuch as only the preceding 28 days of load data are used to forecast the following 24-hour load pattern for forecasts performed on Monday through Thursday. On Friday of any given week, forecasts for the following Saturday through Monday include 72 hours of forecasted loads. Filtering consists of differencing the load levels, not logarithms. The data were differenced at a one-hour lag, a 24hour lag, and a combined 1- and 24-hour lag. For illustrative purposes, the 28-day period ending June 7, 2005 was chosen for examining the effects of filtering. Other periods during the year gave similar results.





Figure 4b. ACF, 24-Hour Differences, 28-Day Period, May 11 - June 7, 2005



Figure 4c. ACF, 1- and 24-Hour Differences, 28-Day Period, May 11 - June 7, 2005



With either one-hour or 24-hour differencing, the autocorrelation function of the differenced series display pronounced "seasonality" with a strong autoregressive relationship (see figures 4a & 4b). But when load levels are differenced both at 1- and 24-hour lags, the strong AR relationship in the autocorrelation function is attenuated (figure 4c).

The autocorrelation function in figure 4c displays almost no discernible trends. None of the autocorrelations for more than 24-hour lags exceed twice the estimated standard errors.

The partial autocorrelations for the 1- and 24-differenced series display an oscillatory decaying pattern at roughly 26, 50, 74, . . . hours (see darker columns in figure 5). This pattern may suggest some residual moving-average relationships not captured by merely differencing the hourly load series.





Using the filtered data, various procedures can be used to identify the appropriate ARIMA model. The most common procedures favor a parsimonious model, that is, as model in which a smaller number of parameters is preferred to one with more parameters. Information criteria such as Akaike's or Schwartz's criteria are typically used to select the most parsimonious model. Alternatively, Lagrange multiplier (LM) tests for determining the appropriate ARMA specification have been proposed (Godfrey; Hall and McAleer). For brevity, the results of information criteria and LM tests are not presented here.

The multiplicative seasonal ARIMA model (see Hagan and Behr) chosen for analysis is given as

$$(1 - a_1 L^1 - a_2 L^2) (1 - a_{168} L^{168}) \Delta^1 \Delta^{24} y_t = (1 + b_{24} L^{24} + b_{48} L^{48}) \varepsilon_{48} L^{48}$$

There are five parameters to estimate for any particular rolling sample. No intervention variables are used nor are weather variables included in a transfer function.

Estimation. The parameters were estimated by maximum likelihood in nearly all cases. On occasion, maximum likelihood fails to converge in which case the conditional non-linear least squares was used to obtain parameter estimates. Estimation of the model with a sample of 672 hourly observations takes less than one second of cpu using *SAS* version 9.1 software on a typical personal computer (see table A1, page 46, for typical estimation results). Forecasted load values can be written directly to *Excel* spreadsheets for subsequent graphing and analysis. *Forecasting*. Forecasted load values were obtained in the usual fashion. Although confidence intervals about forecasted values could be generated, they are not included here.

Seasonal Forecasting

All forecasts discussed in this section apply to the total of all 6 Class A members. The ARIMA model just mentioned could be estimated with ease every day or even more frequently to obtain new parameter estimates and, consequently, updated forecasts of next-day 24-hour load profiles. Reporting the results of 260 week-day forecasts for an entire year would be

overwhelming, however. Instead, five typical seasons of the year were identified and the forecasting results from each of these five weeks are reported in what follows.

Although there may be various meteorological criteria for identifying particular seasons as distinct from one another, we choose to identify separate seasons based on the shape of daily load profiles only. Even though load profiles for each Class A member may differ from the total or aggregate load profiles, any disaggregated differences are ignored at this juncture. The five seasons are given in table 1 below.

Season	Type of 24-Hour Profile	Week of Analysis
Winter	Bimodal	January 27 – February 2
Spring	Transitional: Mixture of Bimodal and Unimodal	March 30 – April 5
Summer, Pre-Monsoon	Unimodal	June 8 - 14
Summer, Monsoon	Unimodal; Highly variable peak due to rain events	August 3 – 9
Fall	Transitional: Mixture of Bimodal and Unimodal	October 26 – November 1

 Table 1. Seasons for Forecasting, Calendar Year 2004

The terms unimodal and bimodal refer to whether the daily profile has one or two peaks. While winter profiles are nearly always bimodal and summer profiles are unimodal, spring and fall profiles often display shapes which are a mixture of both.

Using rolling samples of 672 hourly observations, the ARIMA model was estimated for each week day in each of the five weeks selected in 2004, resulting in 25 sets of forecasted values. Forecasted loads were compared to actual loads visually as well as using several metrics. The most widely used metric in the academic literature is the mean absolute percentage error (MAPE) defined as

$$MAPE = \frac{1}{T} \sum_{t=1}^{T} \frac{|y_t - \hat{y}_t|}{y_t}$$

where y_t is the actual load level at any given hour, \hat{y}_t is the forecasted load value for that same hour, and *T* is the total sample size (i.e. number of hours) over which the average is calculated. As may be obvious, MAPE treats the magnitudes of over and under forecasting equally. Squaring the forecast error, instead of taking the absolute value, would add a larger penalty for larger misses whether they were positive or negative. A mean squared percentage error criterion such as

MSPE =
$$\frac{1}{T} \sum_{t=1}^{T} \frac{(y_t - \hat{y}_t)^2}{y_t}$$

might be used if analysts think that larger forecasting errors should be weighted more heavily. In the following sections, only MAPE are reported.

As objective 1 of this project states, forecasting the level and hour at which the coincidental peak load occurs is extremely important. Accordingly, we report the absolute magnitude of forecast errors as well as percentage errors. Also, we report the time of day of actual coincidental peaks versus forecasted values (see Tables A1-A5, pages 45-47).

Peak Forecasts: Weekdays

Objective 1 of this project calls for forecasts of coincidental peak load. The multiplicative ARIMA model gives reasonable estimates of the magnitude and time of day of peak loads (Tables A1-A5). Considering all next-day forecast, as opposed to 72-hour forecasts on Friday of each week, forecasts errors are usually at or below 5 percent regardless of season. There is one notable exception: the forecast error for Wednesday, June 9 was 18.2 percent,

corresponding to a forecast of 62.2 MW over the actual load. Daily maximum temperatures in one of the six Class A member areas—Trico—dropped dramatically relative to typical daily high temperatures (see figure 6). Although these temperatures are only indicative of a single member area, they suggest a plausible reason for the high forecast values: load owing to peak air conditioning demands dropped as daily maximum temperatures fell. Interestingly, however, for the following day, Thursday, June 10, the forecast peak load "recuperated," over-forecasting by only 11.0 MW or 3.3 percent even as the daily maximum temperature in Tucson fell to a near record low.



Figure 6. Hourly Dry Bulb Temperatures, June 1 -14, 2004, Tucson International Airport

The forecasted time of day of coincidental peaks is generally good. When daily profiles are unimodal, the hours of coincidental peak is forecasted exactly in nearly all cases. In the summer when peak loads are highest, the only misses were by one hour. When daily profiles are bimodal, however, forecasting the exact time of day of coincidental peak load can become more difficult because morning and evening peaks may occur at nearly the same load levels. On Saturday, January 31, for example, the peak load occurred at 8 p.m. whereas on the previous four days, the peak occurred at 8 a.m. Not surprisingly, the ARIMA model forecast an 8 a.m. peak

Day in June

for Saturday. The magnitude of the error was small because the actual Saturday evening peak load was only 6 MW larger than the peak morning load that same day.

Peak Forecasts: "Weekends"

Forecasts of daily coincidental peak loads for the "weekend" of Saturday, Sunday, and Monday are less accurate as would be expected when the length of the forecast period is tripled from 24 to 72 hours. With a few exceptions, the percentage errors on peak load for Sundays and Mondays are almost uniformly higher than for corresponding Saturdays (see figure 7 below).



Figure 7. Coincidental Peak Forecast Errors, Weekends

During the winter, spring, and fall weeks, the larger forecast errors over the weekend result in errors no larger than about 25 MW. But during summer peaks, the magnitudes of forecast errors can double, resulting in peak forecast errors of over 50 MW (e.g. Monday, June 14, 2004).

Forecasting Daily Load Profiles

Two measures are used to judge the suitability of forecasts for capturing the shape of daily load profiles: visual inspection of graphs and analysis of forecasting errors. Visual inspection is appealing because our eyes tend to see deviations from patterns that do not reveal themselves as easily in tables of numbers. But visual inspection is not without problems: occasionally our eyes can fool us because of optical illusions. Consequently, various analyses of forecasting errors are graphed and analyzed to supplement visual inspections of 24-hour load profiles. For purposes of comparison, forecasts of load profiles are divided into three categories: (i) unimodal summer; (ii) winter bimodal; and (iii) spring and fall transitional loads. Each type of load profile presents special challenges for forecasting.

Unimodal Summer Profiles

Forecasting the time of day of peak loads with unimodal daily load profiles is often the "easiest" in the sense that the ARIMA models provide accurate forecasts. But even small percentage errors in summer forecasts represent very much larger loads. Accordingly, if ARIMA forecasts do not coincide very closely with actual daily load profiles in the summer, cumulative misses over a 24-hour period can be quite large.

To appreciate how large errors of summer loads can be, it is useful to compare two days in the pre-monsoon season in June 2004. As was made clear in the previous section, Wednesday June 9 was a difficult day to forecast: the peak was forecast too high by 62.2 MW (18.2%). From figure 8 it is also clear the shape of the forecast load profile matches the actual load until 8 a.m. when it begins to diverge considerably throughout the rest of day (forecasted values are indicated by dashed lines). By contrast, two days later on Friday June 11, the forecast load profile nearly coincides with the actual loads at every hour. Even the very small "shoulder" occurring from 8:00 to 9:00 p.m. is mimicked by the forecasted values.

Percentage errors for the daily load profiles for all days of the week of June 8 -14 are depicted in figure 10. The largest forecast errors measured in percentage terms sometimes occurred in the neighborhood of peak hours. In contrast, percentage forecast errors at overnight low load levels tend to be quite small. The largest daily percentage errors in hourly forecasts during this week never occurred at peak load hours (see times of day in figure 10).



Figure 8. Load Profiles, June 9, 2004



Figure 10. Hourly Percentage Forecast Errors, June 8 – 14, 2004

In general, actual load profiles are reasonably well approximated when forecasts are made for the next 24 hours. The shape of weekend forecasts tends to diverge over the longer 72-hour forecasting period. For the weekend of Saturday June 12 through Monday June 14, the tendency of the ARIMA forecast to under-predict is exacerbated by Monday (figure 11).



Here again, the cost of forecasting 72 hours ahead rather than 24 hours ahead on each Friday is a deterioration in the accuracy for forecasts.

Daily profiles for the monsoon week chosen are quite similar to those in the pre-monsoon week. The percentage forecast errors were smaller with only 3 of the 168 hours exceeding 10% in absolute value.

Bimodal Winter Profiles

As might be expected, forecasting the load shape for a bimodal daily load profile is more difficult because there are more turning points. Further, forecasting the profile of the daily peak is not easy when morning and afternoon peaks are nearly of equal load. Again, it is useful to contrast a relatively good forecast of the load shape with one that diverges, all within the same week. A relatively good forecast of load occurred for Wednesday, January 28 when both peaks are very closely approximated (see figure 12 below). However, the mid-afternoon trough is forecast about 10 percent too high.





The following Saturday, January 31 was the day when the peak was forecast for the morning even though it actually happened in the afternoon. As is evident if figure 13 below, the forecasted load profile is too high until 9 a.m. and then consistently too low for the rest of the day.





Despite the difficulty of forecasting the bimodal load profiles, hourly percentage forecast errors were quite reasonable. Figure 14 below shows that most errors ranged from -10% to 1% except for the weekend forecasts when Monday's errors exceeded -10 percent. Perhaps coincidentally, the largest percentage errors in the day-ahead forecasts tended to occur in the vicinity of the afternoon peak from 4 p.m. to 7 p.m. Due to generally lower load levels in the fall, the magnitude of the forecast errors never amounted to more than 20 MW except at 5 a.m. on Monday in the weekend forecast resulting in a 27 MW shortfall.



Figure 14. Hourly Percentage Forecast Errors, January 27 – February 4, 2004

Spring and Fall Transitional Profiles

Transitional profiles in the spring and fall tend to combine elements of unimodal and bimodal profiles depending on weather conditions: the more summer-like the weather the closer to unimodal; the more winter-like, the closer to bimodal. The shapes of spring and fall profiles tend to look indistinguishable in some cases, suggesting there are not distinct spring and fall profiles per se. Nevertheless, both spring and fall profiles will be discussed in what follows.

ARIMA forecasts of the next 24-hour profile sometimes predict the peak level and time of day quite accurately yet fail to track the shape of the entire profile. An example of this phenomenon is exemplified in figure 15. The April 2 peak was forecast only 3.6 MW (1.6%) too high and correctly at 8:00 p.m. However, the forecast profile misses the mid-day trough by as much as 10 percent. On the other hand, for some days, the ARIMA forecast not only hits the peak correctly but follows the shape of the load profile very closely. Figure 16 displays one such case in which the peak was under-forecast by only 0.7 MW (-0.3%) and correctly at 7:00 p.m. The largest hourly error in forecast on that Tuesday was only -2.4% at 1 p.m. These two days



Figure 15. Load Profiles, Friday, April 2, 2004

Figure 16. Load Profiles, Tuesday, October 26, 2004



are indicative of the extremes in performance of the 24-hour ahead ARIMA forecasts of transitional profiles: Friday April 2 is among the worst performances and Tuesday October 26 was one of the best.

Hourly forecast errors during both the spring and fall transitional weeks are depicted below. In the spring week chosen, the largest percentage errors tend to occur in the afternoon



Figure 17. Hourly Percentage Forecast Errors, March 30 – April 5, 2004

Figure 18. Hourly Percentage Forecast Errors, October 26 – November 1, 2004



but not during peak load periods of 8:00 p.m. For the fall week, there is no apparent pattern to the timing of the largest percentage errors. As is evident in other seasons of the year, Friday forecasts of 72-hour ahead loads result in consistently larger percentage forecast errors.

Improving Weekend Forecasts

As an experiment to see how weekend forecasts might be improved, the ARIMA model was estimated with rolling samples ending on a Saturday and Sunday (April 3 and 4, 2004) to see how forecasts would change if the ARIMA parameter estimates were updated daily rather than using the previous Friday's parameter estimates to forecast for the following 72-hour period. In what follows, the forecasts based on Friday's parameter estimates will be referred to as 3-day-ahead forecasts while the forecasts.

Table 2 below indicates that there could be significant reductions in forecast errors if dayahead forecasts were implemented. Although the 3-day-ahead and day-ahead forecasts both

Period	Peak 1	Error	Peak 9	% Error	MAPE		
	3-Day Ahead	Day Ahead	3-Day Ahead	Day Ahead	3-Day Ahead	Day Ahead	
Sunday	14.2	8.4	6.3%	3.7%	7.9%	6.3%	
Monday	18.4	4.8	8.1%	2.1%	9.0%	3.8%	
Sunday-Monday					8.4%	5.1%	

Table 2. Day-Ahead vs. Day-Ahead Forecast Errors, April 4 – 5, 2004

predicted the correct time of day of the peak—8:00 p.m. on Sunday and Monday—the forecasted loads differ substantially as do the percentage errors at peak. The MAPE for each day is also

reduced appreciably using the day-ahead forecasts. The improvement owing to day-ahead forecasts can be appreciated visually in the figures below. Both the day-ahead and 3-day-ahead



Figure 19. Load Profiles and Forecasts, April 3-4, 2004

Figure 20. Hourly Forecast Errors, April 3-4, 2004



forecasts miss the slight morning peak on Monday, April 4. But the day-ahead forecast misses the mild mid-day trough by much less and more closely approximates the evening peak. The day-ahead forecasts also do much better at predicting overnight minimum loads.

The foregoing experiment suggests there may be significant reductions in forecast errors if weekend forecasts can be made on Saturday and Sunday. Further, experimentation with other weeks during the year is recommended so as to quantify the gains when other types of load profiles are prevalent.

Comparing AEPCO and UofA Forecasts

Whether the ARIMA forecasts developed by the UofA can complement AEPCO forecasts depends in large measure whether UofA forecasts can consistently match or better AEPCO forecasts. Both forecasts were compared for two selected weeks, one in the premonsoon season and one in the monsoon season of summer 2005. One of the six Class A members, Mohave Electrical Cooperative (MEC), is a partial requirements member which means the pre-planning unit at AEPCO does not forecast their entire load. As a result, all forecasts and actual loads presented here are for the other 5 Class A members, exclusive of MEC.

Peak Load Comparisons

Comparisons of performance for forecasting peak loads are made in tables 3 and 4 below. The time of day of coincidental peak was predicted correctly for all days by the UofA ARIMA forecasts for the week of June 7 – 13, 2005. The AEPCO time of day peak was only incorrect for Sunday; AEPCO forecast the peak at 3:00 p.m. but it actually happened at 5:00 p.m. In general, the magnitudes of the UofA forecast errors compare favorably with those of AEPCO. In percentage terms, the UofA errors are as small or smaller than AEPCO forecast errors at the

peak for most days of the week. Both AEPCO and UofA forecast reasonably well during this particular week of June 2005 because the weather was not anomalous and the single peaks occurred at 5:00 p.m. every day of the week. In relative terms, this was an "easy" week for forecasting.

Day of the Week	Actual Peak	Forec	ast	Error		Percentage Error	
	Load (MW)	AEPCO	UofA	AEPCO	UofA	AEPCO	UofA
Tuesday	241.0	227	236.3	-14.0	-4.8	-5.8%	-2.0%
Wednesday	247.4	248	237.6	0.6	-9.8	0.2%	-4.0%
Thursday	242.3	246	242.2	3.7	-0.1	1.5%	0.03%
Friday	240.9	256	243.1	15.1	2.1	6.3%	0.9%
Saturday	239.2	256	246.9	16.8	7.7	7.0%	3.2%
Sunday	249.8	270	249.8	20.2	0.02	8.1%	0.01%
Monday	269.5	248	252.0	-21.5	-17.5	-8.0%	-6.5%

Table 3. Peak Load Comparisons, June 7 – 13, 2005

A more difficult week for forecasting occurred in the monsoon season of 2005. The week of August 9 – 15, 2005 proved especially difficult because there was an unexpected widespread rain event with lower than normal temperatures in southwest Arizona. Peak forecasts for most days during that week were reasonably good (see table 4). However, on Sunday, August 14 peak load dropped by 50 - 75 MW compared to previous days as the result of a monsoon rain event on the prior evening and rain showers that morning in Tucson (see figure 22). At Tucson International Airport the temperature dropped from 77° F. at 8:55 a.m. to 67° F. two hours later as the result of overcast skies and scattered rain showers which lasted until noon (see figure 21). The afternoon high temperature that Sunday only reached 78 ° F. as skies cleared. As a result of the monsoon rain event, both UofA and AEPCO forecasts failed

miserably to predict the time and magnitude of the coincidental peak. The actual peak did not occur until 8:00 p.m. at a load level of only 184.5 MW. But both AEPCO and UofA forecast the peak in the afternoon at

Day of the	ay of the Actual		Forecast		Error		Percentage Error	
Week	(MW)	AEPCO	UofA	AEPCO	UofA	AEPCO	UofA	
Tuesday	238.5	286	265.5	47.5	27.0	19.9%	11.3%	
Wednesday	238.2	273	251.3	34.8	13.1	14.6%	5.5%	
Thursday	270.3	246	248.1	-24.3	-22.3	-9.0%	-8.2%	
Friday	256.5	244	266.5	-12.5	10.1	-4.9%	3.9%	
Saturday	233.7	244	240.8	10.3	7.1	4.4%	3.0%	
Sunday	184.5	286	237.3	101.5	52.8	55.0%	28.6%	
Monday	252.9	244	233.3	-8.9	-19.5	-3.5%	-7.7%	

Table 4. Peak Load Comparisons, August 9 – 15, 2005

3:00 and 4:00 p.m., respectively. The AEPCO forecasted peak was 100 MW too high while the UofA peak was over 50 MW too high. Clearly, the unexpected monsoon event rendered both forecasted peaks quite misleading.

The effects of the monsoon event on hourly loads can be appreciated in figure 23. The low temperatures, precipitation, and cloud cover lead to a profile which more closely resembled a typical fall profile than a unimodal summer profile. Yet both AEPCO and UofA forecasts were typical summer forecasts. Neither forecast could accurately predict the timing or magnitude of peak load given the unexpected rain event.

Profile Comparisons

Forecasting coincidental peak loads is indisputably important. But forecast errors at other hours during the day can have big consequences in the summer when loads are at their



Figure 21. Hourly Dry Bulb Temperatures (° F.), Tucson International Airport

Figure 22. Hourly Precipitation (Inches), Tucson International Airport



highest during the year. Hence, the forecast performance measured in terms of tracking load profiles is especially important in the summer.

As a means of comparing forecast performances at all hours, the percentage forecast errors over the course of the selected pre-monsoon and monsoon weeks are presented in figures 24 and 25. For the pre-monsoon week, the UofA forecasts of weekdays—Tuesday through Friday—tend to have slightly smaller percentage errors than the AEPCO forecasts. But both forecasts of weekends—Saturday through Monday—deteriorate relative to weekday forecasts, and there is little appreciable difference in forecast errors between the two forecasts.



Figure 23. Load Profiles, Sunday, August 14, 2005

Figure 24. Hourly Percentage Forecast Errors, June 7 - 13, 2005



For the monsoon week chosen, UofA forecasts again have slightly smaller percentage



Figure 25. Hourly Percentage Forecast Errors, August 9 - 15, 2005

errors but both forecasts failed miserably to capture the effects on load of the unexpected monsoon rain event already discussed (see figure 25 above).

A daily recap of the two forecast errors in table 5 bears out the tendency of UofA forecasts to yield slightly smaller percentage errors for weekday forecast in both weeks. But both sets of forecasts fair much worse in the monsoon week even when there are no significant rain events. That the UofA ARIMA forecasts would be as accurate as they are is surprising inasmuch as the ARIMA model does not use weather data in any fashion. Only past hourly load data are used to make short-term load forecasts. One virtue of the ARIMA model is that it can be estimated quickly based only on previous load data, thereby producing a useful benchmark against which forecasters in the pre-planning unit can compare their forecast.

Time Deried	June 7	- 13	August 9	- 15
	AEPCO	UofA	AEPCO	UofA
Tuesday	3.5%	1.5%	11.1%	5.2%
Wednesday	4.5%	2.6%	8.6%	3.8%
Thursday	2.7%	1.0%	6.3%	5.2%
Friday	4.0%	1.0%	5.6%	5.7%
Saturday	3.8%	2.4%	11.2%	4.0%
Sunday	4.2%	3.8%	39.0%	18.8%
Monday	4.6%	4.5%	14.4%	7.2%
Tuesday - Monday	3.9%	2.4%	13.7%	7.1%

 Table 5. Mean Absolute Percentage Errors, Selected Days and Weeks, 2005

Conclusions

ARIMA models appear to give modest improvements in forecast accuracy compared to status quo methods used for pre-planning purposes, at least using the measures of accuracy utilized in this report. UofA ARIMA models offer improved predictions of coincident peak load and 24-hour load profiles. Further, AEPCO personnel noted the unforeseen benefit of UofA's models: their ability to predict daily *minimum* load levels, which are also important for efficient capacity utilization at the Apache Junction generating plant.

The modest forecasting success achieved with UofA ARIMA models should be taken only as a partial measure of success. UofA and AEPCO forecasts were compared systemically for just two weeks in the summer of 2005. In the future, both sets of forecasts should be compared for the entire summer, and, as other AEPCO forecasts become available, for other times during the year with different load profiles. Although there is no reason to believe ARIMA forecasts will perform worse at other times of the year, careful comparisons of both forecasts should be conducted until analysts are sufficiently convinced that the ARIMA forecasts are sound.

Implementation of ARIMA Forecasts

Implementation of ARIMA forecasts should be relatively easy. Econometricians at AEPCO with ready access to past load data can estimate an ARIMA model in seconds with suitable software. Forecasts can be written easily to spreadsheets for immediate dissemination to analysts elsewhere in AEPCO and Sierra Southwest Cooperative Services. The ease of estimating the ARIMA model suggests the model might be updated as needed and as the most recent hourly load figures become available. For example, the ARIMA model could easily be estimated in the morning for next-day pre-planning purposes but could then be updated late in the afternoon as new load information becomes available.

The ease of estimating the ARIMA model also suggests it could be estimated on Saturdays and Sundays for obtaining next-day forecasts to replace the current 3-day ahead forecasts generated on Fridays. With remote computer access, an AEPCO econometrician could even re-estimate the ARIMA model on Saturdays and Sundays without being physically present in Benson.

Incorporating Weather Variables

The modest success achieved also implies that there is still room for improvement. The ARIMA models employed in the project do not incorporate weather data of any sort. In order to incorporate weather data, there are several issues which need to be addressed:

1. Temporal aggregation: hourly vs. daily weather data

- Spatial aggregation: finding the appropriate weather stations for each Class A member site.
- 3. Selection of appropriate weather variables and how to include them in the transfer function.
- 4. Availability of next-day forecasted weather variables.

Each of these issues will be discussed in turn.

<u>Temporal Aggregation</u>. Because the ARIMA model utilizes hourly data, the most useful kind of weather data would be measured at hourly intervals. Daily averages, minima, or maxima likely do not display sufficient variability relative to hourly load readings to be of much value in ARIMA models. But if hourly data are not available at all sites, experiments with daily weather variables should be pursued.

It is worth emphasizing that long historical series of hourly weather data are not necessary for forecasting purposes. In its current form, the ARIMA model only uses the previous 28 days of load data. Similarly, only the previous 28 days of hourly weather data would be necessary to incorporate into the ARIMA model. This suggests that hourly data from new weather stations could be used as it becomes available.

<u>Spatial Aggregation</u>. Regardless of the observation interval of the weather data—hourly or daily—the question of appropriate location of the weather stations needs attention. Obviously, the location of currently functioning weather stations will determine the available set of possible weather stations. Within any Class A member's service area, there is considerable variation in micro-climates, which suggests fine tuning weather-load relationships might call for data from more than one weather station. When forecasting *coincidental* loads, the question of spatial aggregation again deserves attention. Anza and Mohave Electric Cooperatives are situated in

areas with weather patterns distinct from those found in southwest Arizona. But even across members in southwest Arizona there is considerable variation in weather patterns. How to best incorporate weather variables across such varying areas will be a challenge.

One simple option for incorporating weather variables would be to work on objective 3 of the project, namely, generating forecasts of next-day 24-hour load profiles for each Class A member. Once the individual ARIMA models are estimated with transfer functions including weather variables, the coincidental 24-hour profiles could be generated by adding up the individual Class A member forecasts.

<u>Selection of Weather Variables</u>. Weather variables of possible interest to include in a transfer function are dry bulb temperature, relative humidity, wind speed, rainfall, and sky conditions. The academic literature is rife with different approaches to selecting and using various weather variables. Even when using only temperature, various nonlinear, threshold, and cumulative relationships are posited to account for load-temperature relationships. Figure A1, page 50, indicates the contemporaneous relationship between hourly load and temperature is nonlinear. Choosing weather variables to use and discerning how to use them in transfer functions for models for each Class A member could easily take an entire semester of work.

Forecasted Weather Variables. Although it is obvious to an econometrician, it is worth emphasizing that in order to generate next-day forecasts of load, it is necessary to have *forecasted* values of the weather variables included in the model. Without forecasted weather values, it is impossible to obtain load forecasts. Ideally, weather forecasts would be obtained from a reputable government or commercial source. If hourly weather observations are used for the previous 28 days in the ARIMA-transfer function model, then the forecasted values of the weather variables must also be hourly.

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Appendix

Parameter	Parameter Estimate	Standard Error	t-statistic	p-value
a_1	0.778	0.039	19.89	<.0001
a ₂	-0.142	0.039	-3.66	0.0003
a ₁₆₈	0.051	0.043	1.18	0.2361
b ₂₄	0.475	0.040	11.86	<.0001
b ₄₈	0.148	0.040	3.71	0.0002
Std. Error of Estimate		2.248		
Akaike Information Criterion		2900.2		
Schwarz Baye	esian Criterion	2922.5		

 Table A1. Maximum Likelihood Estimation Results, Multiplicative ARIMA Model

Sample: Hour 1, May 11 – Hour 24 June 7, 2005 (672 observations).

Period		Peak	Levels	Peak Hours			
	Actual	Forecast	Error	Percent Error	Actual	Forecast	MAPE
Tuesday	269.1	266.1	-3.0	-1.1%	8	8	4.7%
Wednesday	258.1	252.2	-5.9	-2.3%	8	8	2.7%
Thursday	259.7	243.1	-16.6	-6.4%	8	8	3.0%
Friday	254.5	252.4	-2.1	-0.8%	8	8	2.2%
Saturday	233.4	232.4	-1.0	-0.4%	20	8	5.1%
Sunday	237.7	234.1	-3.6	-1.5%	21	20	4.0%
Monday	271.7	245.4	-26.3	-9.7%	8	8	8.3%
Week							4.3%

 Table A2. Forecasting Results, January 27 – February 2, 2004

Table A3. Forecasting Results, March 30 – April 5, 2004

Dariad	Peak Levels			Peak Hours		MADE	
renou	Actual	Forecast	Error	Percent Error	Actual	Forecast	MAL
Tuesday	263.9	255.1	-8.8	-3.3%	16	20	2.7%
Wednesday	257.4	260.0	2.6	1.0%	14	20	2.3%
Thursday	237.6	252.4	14.7	6.2%	20	20	4.2%
Friday	226.6	230.2	3.6	1.6%	20	20	4.3%
Saturday	219.0	234.1	15.1	6.9%	20	20	5.5%
Sunday	225.7	239.9	14.2	6.3%	20	20	7.9%
Monday	227.5	245.9	18.4	8.1%	20	20	9.0%
Week							5.1%

Daniad		Peak	c Levels	Peak Hours			
Period	Actual	Forecast	Error	Percent Error	Actual	Forecast	MAPE
Tuesday	392.4	421.7	29.2	7.5%	18	17	5.2%
Wednesday	340.8	403.0	62.2	18.2%	17	17	10.0%
Thursday	338.2	349.2	11.0	3.3%	17	17	4.8%
Friday	363.3	356.8	-6.5	-1.8%	17	17	1.6%
Saturday	374.8	372.5	-2.3	-0.6%	18	17	1.9%
Sunday	405.6	373.4	-32.2	-7.9%	17	17	5.2%
Monday	425.9	373.0	-52.9	-12.4%	17	17	7.0%
Week							5.1%

Table A4. Forecasting Results, June 8 – 14, 2004

Table A5. Forecasting Results, August 3 – 9, 2004

Period		Peak	c Levels	Peak Hours		MADE	
	Actual	Forecast	Error	Percent Error	Actual	Forecast	MAFL
Tuesday	427.8	409.4	-18.4	-4.3%	15	17	2.4%
Wednesday	405.1	409.5	4.5	1.1%	17	16	2.4%
Thursday	403.6	411.5	7.9	2.0%	17	17	3.7%
Friday	405.5	403.4	-2.1	-0.5%	16	17	2.4%
Saturday	417.6	409.7	-7.9	-1.9%	16	16	2.6%
Sunday	438.4	405.8	-32.6	-7.4%	17	16	5.6%
Monday	446.6	406.7	-40.0	-8.9%	16	16	7.5%
Week							3.8%

Period		Peak	c Levels	Peak Hours		MADE	
	Actual	Forecast	Error	Percent Error	Actual	Forecast	WIAFE
Tuesday	225.7	225.0	-0.7	-0.3%	19	19	0.8%
Wednesday	223.5	223.8	0.3	0.2%	19	19	0.9%
Thursday	218.7	212.5	-6.2	-2.8%	19	19	2.3%
Friday	207.1	216.5	9.3	4.5%	19	19	2.7%
Saturday	204.1	214.1	9.9	4.9%	19	19	3.1%
Sunday	200.9	223.0	22.1	11.0%	19	19	6.0%
Monday	232.9	228.7	-4.1	-1.8%	19	19	3.1%
Week							2.7%

Table A6. Forecasting Results, October 26 – November 1, 2004



Figure A1. Hourly Load – Temperature Relationships, Jan. 1, 2004 – May 31, 2005

Note: Temperature is from Tucson International Airport; load is the total for all 6 Class A members.