## Cardon Research Papers

Research Paper
2004-17
September 2004
(revised April 2005)

The University of Arizona is an equal opportunity, affirmative action institution. The University does not discriminate on the basis of race, color, religion, sex, national origin, age, disability, veteran status, or sexual orientation in its programs and activities.

Lester D. Taylor<br>University of Arizona

## An Additive Double-Logarithmic Consumer Demand System

Department of Agricultural and Resource Economics
College of Agriculture and Life Sciences
The University of Arizona
This paper is available online at http://ag.arizona.edu/arec/pubs/workingpapers.html

Copyright ©2004 by the author(s). All rights reserved. Readers may make verbatim copies of this document for noncommercial purposes by any means, provided that this copyright notice appears on all such copies.

Abstract<br>An Additive Double-Logarithmic<br>Consumer Demand System<br>Lester D. Taylor<br>University of Arizona

Despite obvious theoretical shortcomings, the double-logarithmic function, because of ease of estimation and generally superior fit, is often the demand function of choice in applied demand analysis. However, the drawback to double-logarithmic demand functions is that they are not theoretically plausible, in that they are neither consistent with an underlying utility function nor additive (in the sense of satisfying the budget constraint). The purpose of this paper is to introduce a doublelogarithmic demand system that is additive. This is accomplished through an extension of the Houthakker's indirect addilog model that allows for all prices, not just the own-price of a good, to be included in each of the demand functions. The system is applied to a cross-sectional data set consisting of six exhaustive categories of consumption expenditure from the four quarterly BLS consumer expenditure surveys for 1996 augmented with price data collected in quarterly cost-of-living surveys conducted by ACCRA.

I am grateful to Sean McNamara of ACCRA for making EXCEL files of ACCRA surveys available to me and to the Cardon Chair Endowment in the Department of Agricultural and Resource Economics at the University of Arizona for financial support. Construction of data sets and econometric estimation have all been done in SAS.

An Additive Double-Logarithmic
Consumer Demand System

Lester D. Taylor<br>University of Arizona

## I. Introduction

Despite obvious theoretical shortcomings, the double-logarithmic function, because of ease of estimation and generally superior fit, is often the demand function of choice in applied demand analysis. However, the drawback to double-logarithmic demand functions is that they are not theoretically plausible, in that they are neither consistent with an underlying utility function nor additive (in the sense of satisfying the budget constraint). The purpose of this paper is to introduce a double-logarithmic demand system that is additive. This is accomplished through an extension of the indirect addilog model of Houthakker (1960) that allows for all prices, not just the own-price of a good, to be included in each of the demand functions. The system is applied to a cross-sectional data set consisting of six exhaustive categories of consumption expenditure from the four quarterly BLS consumer expenditure surveys for 1996 augmented with price data collected in quarterly cost-of-living surveys conducted by ACCRA. ${ }^{1}$

## II. An Additive Double-Logarithmic Demand System

In his development of the indirect addilog model, Houththakker (1960) employed a mathematical device that enables any non-additive function $\theta_{\mathrm{i}}(\mathrm{y})$ to be made additive in terms of y by the transformation,

$$
\begin{equation*}
g_{i}(y)=\frac{y \theta_{i}(y)}{\sum \theta_{k}(y)}, \tag{1}
\end{equation*}
$$

since $\sum g_{i}(y)=y$. The application of this transformation to the double-logarithmic demand function,

$$
\begin{equation*}
\mathrm{q}_{\mathrm{i}}=A_{i} y^{\beta_{i}} \prod_{j=1}^{n} p^{\gamma_{j}}, \mathrm{i}, \mathrm{j}=1, \ldots, \mathrm{n} \tag{2}
\end{equation*}
$$

[^0]then gives an additive system of functions:
\[

$$
\begin{equation*}
f_{j}(y, p)=\frac{A_{j} y^{\beta_{j}+1} \prod_{k=1}^{n} p^{\gamma_{k}}}{\sum A_{j} y^{\beta_{j}} \prod_{k=1}^{n} p^{\gamma_{k}}}, \quad \mathrm{j}=1, \ldots, \mathrm{n} . \tag{3}
\end{equation*}
$$

\]

The denominator in this expression for $f_{j}(y, p)$ is obviously a very complicated function of prices (p) and income (y), indeed so much so that estimation of the functions directly is pretty much intractable. However, following Houthakker's derivation of the indirect addilog model, the messy denominators can be eliminated through division of $f_{j}(y, p)$ by $f_{i}(y, p)$, so that:

$$
\begin{equation*}
\frac{q_{j}}{q_{i}}=\frac{A_{j} y^{\beta_{j}+1} \Pi p^{\gamma_{k}}}{A_{i} y^{\beta_{i}+1} \Pi p^{\gamma_{k}}} . \tag{4}
\end{equation*}
$$

Upon taking logarithms, this expression then becomes:

$$
\begin{equation*}
\operatorname{lnq}_{\mathrm{j}}-\ln q_{\mathrm{i}}=\mathrm{a}_{\mathrm{ij}}+\left(\beta_{\mathrm{j}}-\beta_{\mathrm{i}}\right) \ln y+\sum\left(\gamma_{\mathrm{jk}}-\gamma_{\mathrm{ik}}\right) \ln p_{\mathrm{k}}, \quad \mathrm{i}, \mathrm{j}, \mathrm{k}=1, \ldots, \mathrm{n}, \mathrm{j} \neq \mathrm{i} \tag{5}
\end{equation*}
$$

where $\mathrm{a}_{\mathrm{ij}}=\ln \mathrm{A}_{\mathrm{j}}-\ln \mathrm{A}_{\mathrm{i}}$.
Expression (5) is thus seen to be consist of $n-1$ double-logarithmic equations, in which the "dependent" variables are logarithmic differences, and the "independent" variables are the logarithms of income and the n prices. The coefficients that are estimated in the these equations are not $\beta_{\mathrm{j}}$ and $\gamma_{\mathrm{jk}}$, but rather $\left(\beta_{\mathrm{j}}-\beta_{\mathrm{i}}\right)$ and $\left(\gamma_{\mathrm{jk}}-\gamma_{\mathrm{ik}}\right)$, which would appear to leave the individual $\beta$ 's and $\gamma$ 's unidentified. However, it will be shown below how, by making use of the additivity constraints on income and price elasticities, unique estimates of these underlying parameters can be obtained.

## III. An Empirical Application

We now turn to an empirical application of the model that has just been developed, using cross-sectional data consisting of observations from the four quarterly BLS consumer expenditure surveys for 1996. Since price information is not collected in the CES surveys, the latter have been joined with prices from cost-of-living surveys that are conducted on a quarterly basis by ACCRA. ${ }^{2}$

[^1]Prices are collected by ACCRA in 320 or so U. S. cities for about 60 items of consumption expenditure, from which city-specific indices can be constructed that can be used to measure price differences both though time for a specific city and across cities at a point in time. From the 60 or so items for which price data are collected, ACCRA constructs indices for six broad categories of expenditure, namely, groceries, shelter, utilities, transportation, health care, and miscellaneous. ${ }^{3}$ In the analyses to follow, the six ACCRA categories are allied with comparable categories in the BLS CES surveys. In particular, the ACCRA category "groceries" is identified with the CES category "food consumed at home", while the other four specific ACCRA categories are identified with CES counterparts of the same name. Finally, the ACCRA miscellaneous category is identified with CES total expenditure minus the sum of expenditures for the first five categories. Since, to protect confidentiality, place of residence in the CES samples is specified only in terms of state and size of urban area, the ACCRA city price indices have had to be aggregated to a state level. Weights used in the aggregation are city population from the U. S. Census of 2000. The consequent state-level price indices are then attached to households in the CES samples according to states of residence. ${ }^{4}$ The resulting data set is one consisting of 8056 household observations with variation in prices as well as income.

This, then, is the data set that model developed in the preceding section is applied to. Specifically, with n equal to 6, expression (5) yields five equations to be estimated. Although the results are independent of the particular category to be "left out", the dependent variables are defined as logarithmic deviations from miscellaneous expenditures (category 6). Each of the five estimating equations has the same set of independent variables, namely, the logarithm of total expenditure, the logarithms of the prices of food consumed at home, housing, utilities, transportation, health care, and miscellaneous expenditures, together with a fairly long list of socio-demographical and regional
190.3 for Philadelphia and 196.3 for San Francisco cannot be interpreted as saying that the allitems CPI was 1.03 percent higher in San Francisco than in Philadelphia, but only that the allitems index in Philadelphia was 190.3 percent higher in October, 2003, than it was during the base years of 1982-1984, and similarly for San Francisco. Thus, the areal price indices that are currently constructed by BLS unfortunately cannot serve the need at hand. However, the results that are obtained using ACCRA prices in this study [and also in Taylor (2004a, 2004b)] suggests that efforts to create appropriate BLS areal price indices would be worthwhile.
${ }^{3}$ The items underlying the six ACCRA categories are given in Part A of the appendix.
${ }^{4}$ While attaching prices from ACCRA surveys to the CES samples in the manner described yields a cross-sectional consumption data set in which both price and income elasticities can be estimated, it is important to keep in mind that any attempt to extract price elasticities from household budget data, not just the present effort, is laden with difficulties. For an enlightened discussion of these difficulties that is as fresh today as when written 50 years ago, see Prais and Houthakker (1955).
variables. ${ }^{5}$ The estimated coefficients and t-ratios for total expenditure and price variables are tabulated in Table 1.

In interpreting the coefficients (and their statistical significance) in this table, it needs to be kept in mind that the coefficients being estimated are deviations from the "left out" category, miscellaneous expenditures. Thus, the fact that all of the coefficients on lntotexp are negative, except for the one on transportation expenditures, implies that the total expenditure elasticities will be largest for miscellaneous and transportation expenditures. The large t-ratios associated with these coefficients in turn imply that differences in the total-expenditure elasticities for each of the goods and miscellaneous expenditures are not only large, but also highly statistically significant.

Table 1

Estimating Equations for Additive Double-Log Model
( t -ratios in parentheses)

| Variable | Food | Shelter | Utilities | Transport. | Healthcare |
| :---: | :---: | :---: | :---: | :---: | :---: |
| lnpfood | $\begin{array}{r} -1.4126 \\ (-1.30) \end{array}$ | $\begin{array}{r} 0.0324 \\ (0.08) \end{array}$ | $\begin{gathered} 0.2317 \\ (0.71) \end{gathered}$ | $\begin{array}{r} -0.3595 \\ (-0.79) \end{array}$ | $\begin{array}{r} -1.2066 \\ (-2.44) \end{array}$ |
| lnphous | $\begin{array}{r} 0.1816 \\ (2.31) \end{array}$ | $\begin{gathered} -0.5929 \\ (-5.85) \end{gathered}$ | $\begin{array}{r} 0.0803 \\ (0.90) \end{array}$ | $\begin{array}{r} 0.0439 \\ (0.39) \end{array}$ | $\begin{gathered} 0.0266 \\ (0.22) \end{gathered}$ |
| lnputil | $\begin{array}{r} 0.0503 \\ (0.48) \end{array}$ | $\begin{gathered} 0.3162 \\ (2.35) \end{gathered}$ | $\begin{array}{r} -0.7500 \\ (-7.04) \end{array}$ | $\begin{array}{r} 0.0155 \\ (0.10) \end{array}$ | $\begin{gathered} 0.2976 \\ (1.84) \end{gathered}$ |
| lnptrans | $\begin{gathered} -0.3366 \\ (-2.87) \end{gathered}$ | $\begin{array}{r} 0.0261 \\ (0.17) \end{array}$ | $\begin{array}{r} -0.3024 \\ (-2.52) \end{array}$ | $\begin{array}{r} -1.3018 \\ (-7.76) \end{array}$ | $\begin{array}{r} -0.0857 \\ (-0.47) \end{array}$ |
| lnphealth | $\begin{gathered} 0.2005 \\ (1.37) \end{gathered}$ | $\begin{array}{r} 0.8588 \\ (4.54) \end{array}$ | $\begin{array}{r} -0.0730 \\ (-0.49) \end{array}$ | $\begin{array}{r} -0.3201 \\ (-1.52) \end{array}$ | $\begin{array}{r} -0.4915 \\ (-2.16) \end{array}$ |
| lnpmisc | $\begin{gathered} 0.3553 \\ (1.13) \end{gathered}$ | $\begin{gathered} -0.7041 \\ (-1.73) \end{gathered}$ | $\begin{gathered} 0.7053 \\ (2.19) \end{gathered}$ | $\begin{array}{r} 2.0341 \\ (4.50) \end{array}$ | $\begin{gathered} 1.0309 \\ (2.10) \end{gathered}$ |
| Intotexp | $\begin{gathered} -0.8184 \\ (-43.58) \end{gathered}$ | $\begin{array}{r} -0.2209 \\ (-9.91) \end{array}$ | $\begin{aligned} & -0.8045 \\ & (-41.86) \end{aligned}$ | $\begin{array}{r} 0.2051 \\ (7.62) \end{array}$ | $\begin{aligned} & -0.6763 \\ & (-23.15) \end{aligned}$ |
| $\mathrm{R}^{2}$ | 0.4127 | 0.1278 | 0.4022 | 0.0473 | 0.3256 |

[^2]Turning next to the individual $\beta^{\prime}$ s and $\gamma$ 's, the six $\beta$ 's are easily obtained from the five coefficients on lny, plus a sixth equation representing the constraint that the budget-share weighted income elasticities sum to 1 . Calculation of the $\gamma^{\prime}$ s, on the other hand, is a bit more complicated. Thirty-six equations are required to solve for them, 30 of which are obviously the equations connecting the $\gamma^{\prime}$ s to the coefficients on $\operatorname{lnp}_{j}-\ln p_{i}$ in the five estimating equations. One would then think that the Hicks-Allen additivity conditions (i.e., that the income and own- and cross-price elasticities sum to 0 for each expenditure category) would provide the additional equations needed for identification. However, this is unfortunately not the case, for when linear relationships embodying these restrictions are included, five of the six price coefficients in the "left-out" equation turn out to be co-linear with the 31 other coefficients. Consequently, in order to achieve identification, I have assumed that the own-price elasticity for food is the negative of food's total expenditure elasticity. This identifies $\gamma_{11}$ (and therefore implicitly $\gamma_{61}$ ). For the remaining identifying restrictions, I have used the five apparent co-linear relationships between $\gamma_{6 \mathrm{k}}$ (for $\mathrm{k}=2, \ldots, 6$ ), $\gamma_{61}$, and $\gamma_{\mathrm{ji}}$ for $\mathrm{j}=1, \ldots, 5$ and $\mathrm{i}=1, \ldots, 6 .{ }^{6}$ The resulting estimates of the $\beta^{\prime} \mathrm{s}, \gamma^{\prime} \mathrm{s}$, and price and income elasticities are tabulated in Tables 2 and 3. ${ }^{7}$

A perennial question in the analysis of budget surveys is the extent to which dynamics are reflected in expenditure data. ${ }^{8}$ If dynamics are absent, then the price and income elasticities that are being estimated here can be interpreted as measuring long-run (or steady-state) values, whereas if dynamics are present the estimates are neither fish nor fowl, in the sense of being neither short-run nor long-run. Following a debate in the 1950's concerning the efficacy of incorporating income elasticities that are extraneously estimated from budget surveys into time-series regressions for estimating price elasticities, the view has pretty much been that the situation with budget data is the

[^3]Table 2

Estimated Parameters
Additive Double-Logarithmic Demand System
CES-ACCRA Surveys 1996

| Category | Parameters |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | food | shelter | $\underline{\text { utilities }}$ | trans. | healthcare | misc. | total expenditure |
| food | -0.4158 | 0.0291 | 0.2285 | -0.3628 | -1.2098 | -0.0033 | 0.4158 |
| housing | -0.0626 | -0.8371 | -0.1718 | -0.2004 | -0.2177 | -0.2443 | 1.0133 |
| utilities | -0.2270 | 0.0389 | -1.0272 | -0.2618 | 0.0203 | -0.2773 | 0.4297 |
| trans. | -0.2922 | 0.0705 | -0.2580 | -1.2574 | -0.0413 | 0.0444 | 1.4392 |
| healthcare | -0.1176 | 0.5407 | -0.3911 | -0.6383 | -0.8096 | -0.3181 | 0.5579 |
| misc. | -0.5038 | -1.5633 | -0.1539 | 1.1743 | 0.1718 | -0.8591 | 1.2341 |

Table 3

Price and Total Expenditure Elasticities
Additive Double-Logarithmic Demand System
CES-ACCRA Surveys 1996
(calculated at sample mean values)

|  | (calculated at sample mean values) |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | food | $\underline{\text { housing }}$ | $\underline{\text { utilities }}$ | $\underline{\text { trans. }}$ | $\underline{\text { healthcare }}$ |  | misc. | (total |
| expenditure |  |  |  |  |  |  |  |  |

former, that is, that short-term dynamics are largely absent, so that the estimates obtained (assuming that models are otherwise properly specified) represent steady-state values. The basis for this argument is that, whereas time-series estimates of price and income elasticities will reflect short-run adjustment to changes in income and prices, cross-section estimates will reflect long-run, steadystate adjustment. ${ }^{9}$ The latter is seen as being the case if households, even though they may be in temporal disequilibrium, are affected equally by cyclical and other time-varying factors. For present purposes, the assumption is that the elasticities represent steady-state values.

Looking first at the price elasticities, we see that all own-price elasticities are negative, two of which (transportation and miscellaneous expenditures) are in the elastic range. Perhaps not surprisingly, the smallest own-price elasticity is the one that was imposed in estimation (as the negative of the total expenditure elasticity) for food. Food is also seen to have relatively small crossprice elasticities. The largest cross-elasticities are for health care, specifically with respect to the prices of food (negative) and miscellaneous expenditures (positive). Transportation expenditures also have large cross-elasticities with respect to these two categories. On the other hand, since the obtaining of cross-price elasticities is relatively novel in applied demand analysis (especially with cross-section data), it is not clear at this point just how seriously the numbers in this table are to be taken, whether they might simply be artifacts of the data arising from the rather limited coverage of the ACCRA price data (particularly in the case of transportation and miscellaneous expenditures), or just what. The only conclusion that maybe is really warranted is that more experience is needed.

Since the model under investigation in this exercise can be viewed as an extension of Houthakker's indirect addilog model so as to include prices of all goods in each demand function, it accordingly is of interest to see how the two models compare. The price and total-expenditure elasticities for the indirect addilog model are given in Table $4 .{ }^{10}$ Since the indirect addilog model entails rather strong separability assumptions, the only cross-price effects recorded are those arising from pure income effects, hence the rather small (and negative) "off-diagonal" terms in this table. Accordingly, the only comparisons of real interest are the own-price and total-expenditure elasticities

[^4]Table 4
Price and Total Expenditure Elasticities
Indirect Addilog Model
CES-ACCRA Surveys 1996

|  | (calculated at sample mean values) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | food | shelter | $\underline{\text { utilities }}$ | trans. | healthcare | misc. | total expenditure |
| food | -0.1084 | -0.1204 | -0.1204 | -0.1204 | -0.1204 | 0.1204 | 0.4406 |
| shelter | -0.1656 | -0.6647 | -0.1656 | -0.1656 | -0.1656 | -0.1656 | 0.9518 |
| utilities | -0.0729 | -0.0729 | -0.1530 | -0.0729 | -0.0729 | -0.0729 | 0.5327 |
| trans. | -0.0126 | -0.0126 | -0.0126 | -0.9338 | -0.0126 | -0.0126 | 1.3738 |
| healthcare | -0.0524 | -0.0524 | -0.0524 | -0.0524 | -0.2284 | -0.0524 | 0.6287 |
| misc. | -0.0289 | -0.0289 | -0.0289 | -0.0289 | -0.0289 | -0.9124 | 1.3361 |

Table 4

# Own-Price and Total Expenditure Elasticities <br> Addilog and Double-Log Models <br> CES-ACCRA Surveys 1996 

(calculated at sample mean values)

| Category | Own-Price Elasticities |  |  | Total-Expenditure Elasticities |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Indirect <br> Addilog | Additive Double-Log | Non-Additive Double-Log | Indirect <br> Addilog | Additive Double-Log | Non-Additive Double-Log |
| food | -0.1084 | -0.4158 | -0.7020 | 0.4406 | 0.4158 | 0.3069 |
| housing | -0.6647 | -0.5085 | -0.6153 | 0.9518 | 1.0133 | 0.9088 |
| utilities | -0.1530 | -0.7777 | -0.8937 | 0.5327 | 0.4297 | 0.3154 |
| trans. | -0.9338 | -1.0312 | -1.0777 | 1.3738 | 1.4392 | 1.3224 |
| healthcare | -0.2284 | -0.5040 | -1.1617 | 0.6287 | 0.5579 | 0.4484 |
| misc. | -0.9124 | -0.3478 | -1.1109 | 1.3361 | 1.2341 | 1.1217 |

from the two models. These are tabulated in Table 5. As an added comparison, elasticities from simple (i.e., non-additive) double-logarithmic equations are included as well. ${ }^{11}$

For the own-price elasticities, we see that the largest discrepancies in the three models are for food consumed at home, utilities, and healthcare. Otherwise, orders of magnitudes are pretty much the same. The elasticities for food and utilities are much smaller (in absolute value) in the indirect addilog model than in the double-log models. Because of the strong separability embodied in the indirect addilog model, together with the fact that the non-additive double-log equations have been estimated with only the own-price (which obviously implies a crude informal separability), we should probably expect the elasticities for the indirect addilog model to be in closer agreement with the non-additive model than for the additive and non-additve double-log models. However, this is not the case, for the two double-log models in general yield comparable values. For the totalexpenditure elasticities, agreement across the three models is in general quite close, and, indeed, is extremely so for the indirect addilog and additive double-log models.

## IV. Conclusions

The focus in this paper has been on the development of an additive system of doublelogarithmic demand functions. The model takes its cue from the indirect addilog model of Houthakker (1960), using a mathematical device (apparently first employed by Houthakker) that allows for any non-additive function to be transformed into an additive one. The resulting system of equations, in which demand for each good is a function of every price, satisfy the aggregation condition imposed by the budget constraint that the budget-share income elasticities sum to 1 . However, unlike Houthakker's indirect addilog model, of which the demand system of this paper can be viewed as an extension, the demand equations of the system do not appear to be derivable from an underlying utility function, and thus are not integrable. ${ }^{12}$

The additive double-log equations have been applied to a cross-sectional data set consisting of six exhaustive categories of consumption expenditure from the four quarterly BLS consumer expenditure surveys for 1996 augmented with price data collected in quarterly cost-of-living surveys conducted by ACCRA. Own-price elasticities are all negative, and range from -0.42 for food consumed at home to -1.03 for transportation expenditures. Total-expenditure elasticities, on the other hand, vary from 0.42 for food (in keeping with Engel's Law) to 1.44 for transportation. Cross-

[^5]price elasticities suggest complex patterns of substitution and complementation, in that consumption of many goods appears to be complementary with respect to the price of some other good, but consumption of that other good appears to be a substitute with respect to the price of the first good. Indeed, such asymmetric substitution structures appear almost to be the norm.

In view of the long-standing popularity of double-log functions in applied demand analysis, the demand system of this paper, which is logarithmic and additive and allows for the demand for each good to be a function of all prices, would appear to be a useful addition to the toolkits of applied demand analysis. The system is reasonably straightforward to apply, and, at least in the application here, appears to give plausible results. While the system is not integrable, this, at least in my opinion, is a small price to pay to be able to work with a set of double-logarithmic demand functions that are additive.

## REFERENCES

Houthakker, H.S. (1960), "Additive Preferences," Econometrica, Vol. 28, pp. 244-257.
Houthakker, H. S. and Taylor, L. D. (1970), Consumer Demand in the United States (Second Edition), Harvard University Press.

Phlips, L. (1983), Applied Demand Analysis (Second Edition), North Holland Publishing Co.
Prais, S. J. and Houthakker, H. S. (1955), The Analysis of Family Budgets, Cambridge University Press.

Taylor, L. D. (2004a), "Price and Income Elasticities Estimated From BLS Consumer Expenditure Surveys and ACCRA Price Data," Department of Agricultural and Resource Economics, University of Arizona, Tucson, Arizona.

Taylor, L. D. (2004b), "Estimation of Theoretically Plausible Demand Functions From U. S. From U. S. Consumer Expenditure Survey Data, Department of Agricultural and Resource Economics, University of Arizona, Tucson, Arizona.

Wold, H. and Jureen, L. (1953), Demand Analysis, John Wiley and Sons.

Appendix
A. Consumption Expenditure Categories Included in ACCRA Price Surveys.

| Groceries Housing | Utilities | Transportation | Health Care | Miscellaneous |
| :---: | :---: | :---: | :---: | :---: |
| t-bone stk. apt. rent <br> gd. beef home price <br> sausage mortgage rate <br> fry chicken home $\mathrm{P}+\mathrm{I}$ <br> tuna  <br> gal. milk  <br> dz. eggs  <br> margarine  <br> parmesan cheese  <br> potatoes  <br> bananas  <br> lettuce  <br> bread  <br> cigarettes  <br> coffee  <br> sugar  <br> cereal  <br> sweet peas  <br> tomatoes  <br> peaches  <br> Kleenex  <br> Cascade  <br> Crisco  <br> orange juice  <br> frozen corn  <br> baby food  <br> Coke  | all electric part electric other energy telephone | bus fare tire bal. gasoline | hosp. room Dr. appt. dentist aspirin | hamburger sand. pizza <br> 2-pc. chicken hair cut beauty salon tooth paste shampoo dry clean men's shirt underwear slacks washer repair newspaper movie bowling tennis balls monopoly set liquor beer wine |

B. Preparation of Data.

The CES quarterly data sets employed in the analysis have been developed from the Public Use Interview Microdata sets for 1996 that are available on CD-ROM from the U.S. Bureau of Labor Statistics. ${ }^{13}$ "Cleansing" of the CES files included elimination of households with reported income of less than $\$ 5000$ and then of households with zero (or negative) expenditures for the commodity category in question. The CES surveys do not include price data. The price data for the analysis are

[^6]taken from the on-going price surveys of the 62 items of consumer expenditure listed in Table A1 above in more than 300 cities in the U.S. that are conducted quarterly by ACCRA ${ }^{14}$. From the 62 items of expenditure, ACCRA constructs six price indices (food, housing, etc.), and then from these an all-items index (which in principle are comparable, on a city basis, to BLS city CPI's). The ACCRA city indices in a state for each quarter are aggregated to the state level using city populations from the US Census of 2000 as weights. ${ }^{15}$ The resulting ACCRA prices are then attached to CES households according to state of residence. ${ }^{16}$
C. Definitions of Variables.

| Infood | logarithm of expenditures for food consumed at home |
| :--- | :--- |
| Inhous | logarithm of housing expenditures |
| Inutil | logarithm of expenditures for household utilities |
| Intrans | logarithm of transportation expenditures |
| Inhealth | logarithm of health care expenditures |
| Inmisc | logarithm of miscellaneous consumption expenditures |
| lnincome | logarithm of household income |
| Intotexp | logarithm of total consumption expenditure |
| Inpfood | logarithm of price index for housing |
| Inphous | logarithm of price index for utility expenditures |
| lnputil | logarithm of price index for transportation expenditures |
| Inptrans | logarithm of price index for health care expenditures |

[^7]| lnpmisc | logarithm of price index for miscellaneous expenditures |
| :--- | :--- |
| lnpall | logarithm of all-items price index |
| no_earnr | number of income earners in household |
| fam_size | size of household |
| age_ref | age of head of household |
| dsinglehh | dummy variable for single household |
| drural | dummy variable for rural area of residence |
| dnochild | dummy variable for children in household under age 4 |
| dchild1 | dummy variable for oldest child in household between |
| dchild4 and at least one child less than 12 |  |


| dwest | dummy variable for residence in west (excluded) |
| :--- | :--- |
| dwhite | dummy variable for white head of household |
| dblack | dummy variable for black head of household |
| dmale | dummy variable for male head of household |
| down | dummy variable for owned home |
| dfdstmps | dummy variable for household receiving food stamps |
| D1, D2, D3, D4 | seasonal quarterly dummy variables. |


[^0]:    ${ }^{1}$ This paper is the third in an on-going investigation into the feasibility of integrating price information into the BLS quarterly consumer expenditure surveys. The first (2004a) analyzes 16 quarters of CES and ACCRA data (1996-1999) using only simple doublelogarithmic equations, while the second (2004b) compares, using only the four quarters for 1996, price and total expenditure elasticities obtained from four "theoretically plausible" demand systems, AIDS, LES, and the Indirect and Direct Addilog models.

[^1]:    ${ }^{2}$ See www.ACCRA.com. For the BLS-CES surveys, the natural place to turn for price data is in the price surveys that the Bureau of Labor Statistics conducts monthly as input into construction of the Consumer Price Indices. Prices for several hundred categories of expenditure for some 140 urban areas are collected in these surveys, so that cross-sectional price variation is in principle available. However, the problem is that indices reflecting areal variation in price levels at a point in time are not currently constructed by BLS, but rather only ones that measure price variation over time. Thus, the facts (say) that the BLS all-items index for October, 2003, is

[^2]:    ${ }^{5}$ A full listing of the variables included in the model is given in Part B of the appendix.

[^3]:    ${ }^{6}$ An unfortunate implication of this procedure is that the resulting price elasticities in conjunction with the already obtained total expenditure elasticities do not satisfy the Hicks-Allen additivity conditions.
    ${ }^{7}$ The elasticities in these tables are calculated according to the following formulae:

    $$
    \begin{aligned}
    \eta_{\text {tot.exp. }} & =\beta_{\mathrm{j}}, \quad \mathrm{j}=1, \ldots, 6 . \\
    \eta_{\text {ownprice }} & =\gamma_{\mathrm{jj}}-\sum \mathrm{w}_{\mathrm{k}} \gamma_{\mathrm{jk}}, \mathrm{k}=1, \ldots, 6 \\
    \eta_{\text {cross-price }} & =\mathrm{w}_{\mathrm{j}} \gamma_{\mathrm{ji}}-\sum \mathrm{w}_{\mathrm{k}} \gamma_{\mathrm{jk}}, \quad \mathrm{i}, \mathrm{k}=1, \ldots, 6, \mathrm{i} \neq \mathrm{j},
    \end{aligned}
    $$

    where $\mathrm{w}_{\mathrm{k}}$ is the budget weight of the $\mathrm{k}^{\text {th }}$ expenditure category. The elasticities, it should be noted, are aligned by column. Hence, the cross-elasticity for food with respect to the price of housing is 0.1816 (not 0.0324 , which is the cross-elasticity of shelter with respect to the price of food).
    ${ }^{8}$ Dynamics, in the sense being considered here, appear to have been first discussed as a problem in the analysis of family budgets by Prais and Houthakker (1955).

[^4]:    ${ }^{9}$ For detailed discussions of the differences between cross-section and time-series estimates of the same parameters, see Kuh and Meyer (1957) and Kuh (1959, 1963).
    ${ }^{10}$ The indirect addilog can be derived either using the procedure employed in deriving the estimating equations for additive double-log model in Section II or else from the indirect utility function,

    $$
    u(y, p)=\sum a_{i}\left(y / p_{i}\right)^{b_{i}}
    $$

    using Roy's Theorem. See Houthakker (1960) or Phlips (1983) for details. For a comparison of the indirect addilog model with the direct addilog model, as well as the Almost-Ideal-DemandSystem and Linear Expenditure System, see Taylor (2004a).

[^5]:    ${ }^{11}$ The non-additive double-log equations include own-price only. When all prices are included, multicollinearity amongst the price variables leads to a positive coefficient for the ownprice in the equation for utility expenditures.
    ${ }^{12}$ Because the Hicks-Allen aggregation conditions are consequences of the budget constraint, rather than of integrability, it would seem that they also ought to be satisfied. However, as has been noted, singularities among the identifying restrictions get in the way of their being imposed.

[^6]:    ${ }^{13}$ See http://www.bls.gov/cex/home.htm.

[^7]:    ${ }^{14}$ See http://www.ACCRA.com.
    ${ }^{15}$ See http://www.census.gov/Press-Release/www/2003/SF4.html
    ${ }^{16}$ In instances in which CES does not code state of residence for reasons of nondisclosure, the households in question are dropped.

