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Internal Structure of U.S. Consumption Expenditures

Lester D. Taylor *University of Arizona*

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Department of Agricultural and Resource Economics College of Agriculture and Life Sciences The University of Arizona

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Lester D. Taylor*
Department of Economics
Department of Agricultural & Resource Economics
University of Arizona

ltaylor@email.arizona.edu

Abstract

Information in a household budget survey is usually analyzed in terms of expenditures for different goods and services and how these relate to income, prices, and socio-demographic factors such as age, family size, and education. Allocation of expenditures amongst different categories of consumption is seen as being determined by tastes and preferences acting in conjunction with a constraint imposed by prices and income. The parameters obtained are obviously useful in analyzing the impact on consumption resulting from changes in income and prices (should the latter be available), but income and price elasticities, in themselves, say little about the internal structure of consumption spending. How expenditures for housing and transportation, for example, are related to expenditures for food has, to the best of my knowledge, never a subject of direct empirical study. The investigation is based upon 40 quarterly surveys of consumer expenditures that were conducted by the U. S. Bureau of Labor Statistics from 1996Q1 through 2005Q4. The focus of the analysis is on two large data sets consisting of coefficients obtained from least-squares regression equations, in which each of 14 categories of expenditure is regressed on the expenditures for the other 13 categories and similarly for budget shares.

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I. INTRODUCTION

Usually, when we consider the information that is given in a household budget survey, we do so in terms of expenditures for different goods and services and how these relate to income, prices, and socio-demographic factors such as age, family size, and education. Allocation of expenditures amongst different categories of consumption is seen as being determined by tastes and preferences acting in conjunction with a constraint imposed by prices and income. The parameters thus obtained are obviously useful in analyzing the impact on consumption resulting from changes in income and prices (should the latter be available), but income and price elasticities, in themselves, say little about the internal structure of consumption spending. How expenditures for housing, transportation, and personal care -- to pick three standard categories of consumption spending -- are related to expenditures for food, for example, has, to the best of my knowledge, never been a direct focus of empirical study. Might there be information lurking in these relationships that provide insight into consumer behavior that complements (or even goes beyond) that given by conventional price and income elasticities? Such is the question that motivates the present study.

The study that follows is based upon the 40 quarterly surveys of consumer expenditures that were conducted by the U. S. Bureau of Labor Statistics from 1996Q1 through 2005Q4. As an illustration of the information that is the focus of the study, Table 1 presents, for the survey from the fourth quarter of 1999, estimated coefficients and t-ratios from a sequence of least-squares regression equations in which each of 14 categories of expenditure is regressed on the expenditures for the other 13 categories. Table 2 presents the same for budget shares. R²s of the equations appear in the last row of the tables.²

Items of note in these tables are as follows:

(1). The categories of expenditure with the "tightest" relationship to their sister expenditures (as measured by R²) are Food, Housing, Apparel, and Personal Insurance. Those with the "loosest" relationship are Transportation, Health, Tobacco, and Contributions.

¹ The traditional approach in demand analysis to questions concerning internal relationships of consumption has been in terms of the functional structure of the utility function, how utilities (or, more specifically, marginal utilities) of individual categories of consumption (whether singly or in groups) might be related to utilities of other categories. In great part, the motivation for such enquiries has been adjunctive to estimation, as demand functions derived from utility functions in which separability is postulated are generally much easier to estimate than demand functions derived from utility functions in which separability is absent.

² All calculations are done in SAS. See Appendix 1 for a listing of the 14 categories of expenditure and their abbreviations.

Table 1

Intra-Budget Regressions

14 Categories of U.S. Consumption Expenditure
1999 Q4

(t-ratios in parentheses)

				Categories			
<u>Coefficients</u>	Food	<u>Alcbev</u>	<u> Housing</u>	<u>Apparel</u>	Trans	<u>Health</u>	<u>Entertn</u>
Intercept	620.57	-1.67	879.77	-43.36	569.82	268.34	-93.37
	(33.67)	(-0.42)	(15.63)	(-3.38)	(5.57)	(15.14)	(-5.32)
Food	- 1	0.0182	0.5009	0.0912	0.2456	0.0739	0.1229
		(6.89)	(13.42)	(10.89)	(3.63)	(6.22)	(10.69)
Alcbev	0.4597	-1	0.9971	0.2411	1.6633	-0.0579	0.3332
	(6.89)		(5.24)	(5.68)	(4.90)	(-0.97)	(5.72)
Housing	0.0618	0.0049	-1	0.0317	0.0878	0.0077	0.0391
	(13.42)	(5.24)		(10.75)	(3.70)	(1.83)	(9.65)
Apparel	0.2259	0.0236	0.6352	-1	0.1445	-0.0410	0.1433
	(10.89)	(5.68)	(10.75)		(1.36)	(-2.19)	(7.89)
Trans	0.0095	0.0236	0.0276	0.0023	-1	0.0056	0.0108
	(3.63)	(4.90)	(3.70)	(1.36)		(2.38)	(4.71)
Health	0.0923	-0.0286	0.0774	-0.0207	0.1798	- 1	0.0658
	(6.22)	(-0.97)	(1.83)	(-2.19)	(2.38)		(5.08)
Entertn	0.1618	0.0173	0.4165	0.0761	0.3649	0.0694	-1
	(10.69)	(5.72)	(9.65)	(7.89)	(4.71)	(5.08)	
Perscare	1.0682	0.0182	3.3659	0.9520	0.6406	0.5988	0.1070
	(8.53)	(0.73)	(9.45)	(12.03)	(0.58)	(5.32)	(0.97)
Reading	0.4349	0.1274	2.4116	1.0121	1.0229	1.0902	1.0262
	(2.81)	(4.15)	(5.49)	(10.38)	(1.30)	(7.91)	(7.64)
Educ	0.0591	-0.0086	0.3330	0.0120	-0.0036	-0.0227	0.0648
	(4.21)	(-3.08)	(8.37)	(1.34)	(-0.05)	(-1.81)	(5.30)
Tobacco	0.3442	0.1237	-0.4039	-0.0531	0.4560	-0.0220	0.1848
	(5.44)	(9.90)	(-2.24)	(-1.32)	(1.42)	(-0.39)	(3.34)
Misc	0.0016	-0.0019	0.8464	-0.0035	-0.1235	0.0056	-0.0361
	(0.15)	(-0.86)	(29.07)	-0.50)	(-2.22)	(0.57)	(-3.78)
Contrib	0.0263	0.0021	0.0262	0.0129	-0.0640	-0.0151	0.0062
	(1.81)	(0.74)	(0.63)	(1.39)	(-0.87)	(-1.16)	(0.49)
Ins	0.1082	0.0080	0.3816	0.135	0.1529	-0.0132	0.0617
	(13.86)	(5.08)	(17.32)	(2.68)	(3.79)	(-1.85)	(8.97)
R ²	0.3459	0.1373	0.4300	0.2620	0.0587	0.0570	0.2668

Table 1 cont'd.

				Categories			
<u>Coefficients</u>	<u>Perscare</u>	Reading	Educ	<u>Tobacco</u>	Misc	<u>Contrib</u>	Ins
Intercept	29.73	8.91	-47.91	67.57	-137.17	-28.78	49.69
	(14.18)	(5.14)	(-2.51)	(16.30)	(-5.61)	(-1.55)	(1.47)
Food	0.0119	0.0032	0.0530	0.0152	0.0024	0.0221	0.3045
	(8.93)	(2.81)	(4.21)	(5.44)	(0.15)	(1.81)	(13.86)
Alcbev	0.0051	0.0239	-0.1953	0.1381	-0.0700	0.0453	0.5694
	(0.73)	(4.15)	(-3.08)	(9.90)	(-0.86)	(0.74)	(5.08)
Housing	0.0046	0.0022	0.0369	-0.0022	0.1541	0.0027	0.1325
	(9.45)	(5.49)	(8.37)	(-2.24)	(29.07)	(0.63)	(17.32)

rel 0.0263	0.0186	0.0267	-0.0058	-0.0128	0.0268	0.0942
(12.03)	(10.38)	(1.34)	(-1.32)	(-0.50)	(1.39)	(2.68)
1.61E-5	0.0003	-0.0001	0.0008	-0.0071	-0.0021	0.0167
(0.58)	(1.30)	(-0.05)	(1.42)	(-2.22)	(-0.87)	(3.79)
th 0.0083	0.0101	-0.0255	-0.0012	0.0104	0.0158	-0.0463
(5.32)	(7.91)	(-1.81)	(-0.39)	(0.57)	(1.16)	(-1.85)
rtn 0.0016	0.0100	0.0765	0.0107	-0.0700	0.0069	0.2283
(0.97)	(7.64)	(5.30)	(3.34)	(-3.78)	(0.49)	(8.97)
care -1	0.0274	0.3732	-0.0944	-0.7786	0.1407	0.4323
	(2.53)	(3.13)	(-3.57)	(-5.10)	(1.22)	(2.04)
ing 0.0414	-1	0.0250	-0.0807	-0.4894	0.3144	1.7707
(2.53)		(0.17)	(-2.48)	(-2.60)	(2.22)	(6.84)
0.0046	0.0002	-1	-0.0069	-0.0646	0.0226	-0.0096
(3.13)	(0.17)		(-2.34)	(-3.79)	(1.75)	(-0.40)
co -0.0239	-0.0135	-0.1403	-1	-0.0130	0.0510	-0.2211
(-3.57)	(-2.48)	(-2.34)		(-0.17)	(0.88)	(-2.08)
-0.0059	-0.0025	-0.0393	-0.0004	- 1	0.0248	-0.1261
(-5.10)	(-2.60)	(-3.79)	(-0.17)		(2.47)	(-6.88)
rib 0.0019	0.0028	0.0241	0.0027	0.0435	-1	0.0832
(1.22)	(2.22)	(1.75)	(0.88)	(2.47)		(3.42)
0.0017	0.0047	-0.0030	-0.0035	-0.0661	0.0249	- 1
(2.04)	(6.84)	(-0.40)	(-2.08)	(-6.88)	(3.42)	
0.1659	0.1664	0.1427	0.0328	0.1691	0.0160	0.2786
i	(12.03) 3	(12.03) (10.38) (1.61E-5 0.0003 (0.58) (1.30) (1.30) (1.30) (1.30) (1.30) (1.30) (1.30) (1.30) (1.30) (1.31	(12.03) (10.38) (1.34) (1.34) (1.61E-5	(12.03) (10.38) (1.34) (-1.32) (1.61E-5	(12.03) (10.38) (1.34) (-1.32) (-0.50) 1.61E-5	(12.03) (10.38) (1.34) (-1.32) (-0.50) (1.39) (1.61E-5

Table 2

Budget Share Regressions

14 Categories of U.S. Consumption Expenditure
1999 Q4

(t-ratios in parentheses)

				Categories			
<u>Coefficients</u>	Food	Alchev	<u>Housing</u>	<u>Apparel</u>	Trans	<u>Health</u>	<u>Entertn</u>
Food	- 1	0.0146	0.7572	0.0538	0.1643	0.1673	0.0436
		(6.22)	(31.69)	(9.79)	(7.61)	(15.79)	(6.77)
Alcbev	0.4670	-1	0.4295	0.0646	0.4505	-0.3144	0.2702
	(6.22)		(2.93)	(2.06)	(3.68)	(-5.14)	(7.42)
Housing	0.1997	0.0354	-1	0.0261	0.0854	0.0567	0.0291
	(31.69)	(2.93)		(9.25)	(7.70)	(10.30)	(8.84)
Apparel	0.3106	0.0117	0.5721	-1	0.1772	-0.1082	0.1593
	(9.79)	(2.06)	(9.25)		(3.40)	(-4.17)	(10.35)
Trans	0.0620	0.0053	0.1221	0.0116	- 1	0.0192	0.0220
	(7.61)	(3.68)	(7.70)	(3.40)		(2.89)	(5.56)
Health	0.2532	-0.0149	0.3255	-0.0284	0.0770	- 1	0.0261
	(15.79)	(-5.14)	(10.30)	(-4.17)	(2.89)		(3.28)
Entertn	0.1850	0.0358	0.4693	0.1171	0.2477	0.0732	- 1
	(6.77)	(7.42)	(8.84)	(10.35)	(5.56)	(3.28)	
Perscare	0.6256	0.0172	1.7814	0.4749	0.3544	0.8491	0.0871
	(14.63)	(0.86)	(8.13)	(10.17)	(1.92)	(9.29)	(1.58)
Reading	0.1866	0.0690	2.0953	0.4211	0.1270	1.5187	0.8201
	(1.14)	(2.40)	(6.62)	(6.22)	(0.48)	(11.59)	(10.45)
Educ	0.0259	-0.0002	0.1122	0.0616	0.0535	-0.0542	0.0723
	(0.96)	(-0.05)	(2.14)	(5.50)	(1.22)	(-2.48)	(5.54)

Tobacco	0.6822	0.0889	0.2773	-0.0141	0.1781	-0.0345	0.0593
	(13.46)	(9.86)	(2.77)	(-0.66)	(2.13)	(-0.08)	(2.38)
Misc	0.0344	0.0017	0.2705	0.0169	0.1114	0.0764	0.0382
	(0.93)	(0.27)	(3.74)	(1.09)	(1.84)	(2.53)	(2.12)
Contrib	0.0648	-0.0019	-0.0251	0.0016	0.0309	0.0881	0.0045
	(1.61)	(-0.27)	(-0.32)	(0.10)	(0.47)	(2.70)	(0.23)
Ins	0.1367	0.0120	0.4768	0.3340	0.2185	-0.0710	0.0512
	(10.29)	(5.11)	(18.81)	(6.10)	(10.09)	(-6.73)	(7.90)
R^2	0.7076	0.1993	0.7018	0.4178	0.3280	0.3527	0.4370

Table 2 cont'd

				Categories			
<u>Coefficients</u>	<u>Perscare</u>	<u>Reading</u>	Educ	<u>Tobacco</u>	Misc	<u>Contrib</u>	<u>Ins</u>
Food	0.0225	0.0124	0.0063	0.0457	0.0044	0.0071	0.1348
	(14.63)	(1.14)	(0.96)	(13.46)	(0.93)	(1.61)	(10.29)
Alcbev	0.0076	0.0147	-0.0018	0.1906	0.0072	-0.0067	0.3815
	(0.86)	(2.40)	(-0.05)	(9.86)	(0.27)	(-0.27)	(5.11)
Housing	0.0065	0.0037	0.0072	0.0049	0.0092	-0.0007	0.1239
	(8.13)	(6.62)	(2.14)	(2.77)	(3.74)	(-0.32)	(18.81)
Apparel	0.0380	0.1620	0.0087	-0.0054	0.0126	0.0010	0.1933
	(10.17)	(6.22)	(5.50)	(-0.66)	(1.09)	(0.10)	(6.10)
Trans	0.0185	0.0003	0.0049	0.0045	0.0054	0.0013	0.0812
	(1.92)	(0.48)	(1.22)	(2.13)	(1.84)	(0.47)	(10.09)
Health	0.0178	0.153	-0.0200	-0.0004	0.0149	0.0146	-0.1059
	(9.29)	(11.59)	(-2.48)	(-0.08)	(2.53)	(2.70)	(-6.53)
Entertn	0.0051	0.0232	0.0749	0.0167	0.0209	0.0021	0.2142
	(1.58)	(10.45)	(5.54)	(2.38)	(2.12)	(0.23)	(7.90)
Perscare	-1	0.0468	0.0058	-0.1254	0.0068	0.0529	0.1686
		(5.07)	(0.10)	(-4.29)	(0.17)	(1.41)	(1.50)
Reading	0.0971	-1	-0.0024	-0.0521	0.0871	0.1544	0.3927
	(5.07)		(-0.03)	(-1.23)	(1.49)	(2.86)	(2.43)
Educ	0.0003	-0.0001	-1	-0.0150	-0.0012	0.0221	0.0445
	(0.10)	(-0.03)		(-2.15)	(-0.12)	(2.48)	(1.67)
Tobacco	-0.0259	-0.0052	-0.0548	-1	-0.0130	-0.0062	0.0378
	(-4.29)	(-1.23)	(-2.15)		(-0.70)	(-0.36)	(0.74)
Misc	0.0007	0.0045	-0.0022	-0.0068	-1	0.0592	0.0639
	(0.17)	(1.49)	(-0.12)	(-0.70)		(4.83)	(1.73)
Contrib	0.0067	0.0094	0.0493	-0.0038	0.0695	- 1	0.1382
	(1.41)	(2.86)	(2.48)	(-0.36)	(4.83)		(3.46)
Ins	0.0024	0.0027	0.0066	0.0026	0.0084	0.0154	- 1
	(1.50)	(2.43)	(1.67)	(0.74)	(1.73)	(3.46)	
R^2	0.4040	0.2874	0.0535	0.1691	0.0577	0.0369	0.4809

- (2). While most coefficients in the equation for a category are positive, a significant number in Education, Tobacco, and Miscellaneous are negative.
- (3). Large t-ratios are common, especially for the coefficients on Food, Housing, Apparel, and Personal Insurance.

Why, some might ask, is an exercise such as this of interest? The answer lies in the fact that,

ex post, the combined effects of income, prices, and tastes and preferences are represented both in the magnitudes of individual expenditures and in the relationships of expenditures to one another, including all cross-price effects. In addition, any "food", "housing", "transportation", etc. influences with causal consequences independent of income and prices will be reflected as well. Why, for example, should t-ratios for the coefficients on Food and Housing generally be larger than on other categories? Might this be a reflection of some sort of "food" and "housing" effects? Unlike in the conventional theory of consumer behavior, which sees interrelations amongst goods and services in terms of prices mediated through tastes and preferences, reference here is to direct actions of expenditures on one another. Some of these may be causal, in the sense that a desire to consume a good or service leads to the consumption of another, while others may take the form of mutual interaction. The need for energy as input into services provided by durable goods exemplifies the former, while food and wine at a dinner party is an instance of the latter.

The plan and methodology of the investigation is as follows. Least-squares equations like the ones in Tables 1 and 2 have been estimated for each of the quarterly BLS surveys from 1996 through 2005.³ The result is the creation of two large data sets consisting of 7840 (40x14x14) intrabudget coefficients and 7280 (40x14x13) budget-share coefficients with their associated t-ratios and R²s.⁴ These two data sets form the raw material for the investigation. The task, to begin with, will be to see how stable the intra-budget and budget-share coefficients are across the surveys, not only from one survey to the next, but for surveys that are separated in time. If the vectors of these coefficients should be stable (either all or just in part), inferences may then be possible regarding the stability of budget allocation decisions, whether with respect to a "representative" household or in terms of the underlying distribution of consumption patterns for the population at large. Inferences may also be possible regarding the existence of "food", "housing", and other effects as noted above. Stability will be the focus of the next two sections and inferences and interpretation in Sections IV - VI.

II. STABILITY OF THE ξ_i VECTORS

Geometrically, the columns in Tables 1 and 2 can be seen as constituting 14 vectors (one for each expenditure category) in a 14-dimensional space. This space will be denoted by Φ and its constituent vectors by $\xi = (\xi_1 \dots \xi_n)$. The premise that underlies the investigation is that, in effect, these vectors, both absolutely and in relation to one another, are a "black box" that incorporates all

³ All data in the study are taken from the public use microdata CD-ROMs of Consumer Expenditure, 1996-2006, obtained from the Bureau of Labor Statistics, U.S. Department of Labor. A detailed description of the surveys can be found in Chapter 4 of Taylor and Houthakker (2010). The sizes of the 40 surveys are given in Appendix 1.

⁴ "Intra-budget" refers to the expenditures equations and "budget-share" to the budget share equations. Intercepts are included in the expenditures equations, but (since budget shares always sum to 1) not in the budget share equations. An interpretation of the intercepts in the expenditures equations will be given in Section V.

of the factors -- income, prices, tastes and preferences, etc. -- that impinge upon the budget behavior postulated in the standard neoclassical assumption of utility maximization subject to a budget constraint. Any effects of the "food" and "housing" type will be represented as well. As an *ex post* picture of the budgeting process, we can imagine the space Φ as a "force field" and the vectors ξ_i representing the "strength" or "charge" of the field at each point in the space. Income can be seen as providing the "energy" to the field, while the ξ_i can be seen as reflecting the conjunctive effects of income, prices, tastes and preferences and other factors. Importantly, while all of the vectors are anchored at -1 on their respective axes, the entire space is infused with "charge", in that the coordinates of the ξ_i can be of either sign. More will be said about this way of viewing the internal structure of budget expenditures in Section IV.

The first test that has been undertaken regarding stability of the ξ_i vectors over the 40 quarters of surveys is a test for "quarter" effects on the elements of ξ_i . This has been done through estimation of regression equations of the form:

(1)
$$b_{ijt} = \alpha_{ij} + \beta_{1ij} + \sum_{s=2}^{40} (\beta_{sij} - \beta_{1ij}) d_{sjt} + u_{ijt}$$
, i, j = 1, ..., 14, t = 1, ..., 40,

where b_{ijt} denotes the jth coefficient for quarter t for the ith expenditure category and d_{sjt} are dummy variables that take the value 1 for s=j and 0 otherwise. The dependent variables accordingly consist of 520 "observations" (13 coefficients for each of 40 quarters).⁷ Note that the left-out dummy variable is the one for the first quarter of 1996. Equation (1) is estimated for each of the 14 categories of consumption. A brief summary of the results is as follows:⁸

- (1). In the equations for expenditures, 14 of the 520 estimated coefficients have t-ratios greater than 2 (in absolute value), which is less than 3 percent. Of these, 10 are in the equation for miscellaneous expenditures. Of the remaining four, three (for food, housing, and personal insurance) are for estimates of the intercept ($\alpha + \beta_1$), while the fourth (for contributions) is for the dummy variable for the first quarter of 2004. Except for miscellaneous expenditures, which is 0.08, R^2 s are less than 0.05, with several less than 0.01.
- (2). In the equations for budget shares, just 7 of the 520 estimated coefficients have t-ratios greater than 2 (in absolute value), in this

⁵ The terms *force*, *field*, *strength*, and *charge* are simply suggestive and intended to be metaphorical.

⁶ The values of -1 for food in the columns for food, for example, are simply a reflection of the fact that food is the dependent variable in these equations.

⁷ The intercepts are excluded in the equations for expenditures.

⁸ Absence of "quarter" effects obviously requires small t-ratios and small R²s.

case all for intercepts (for food, housing, apparel, transportation, personal care, reading, and miscellaneous expenditures). R²s are all less than 0.04, except for 0.08 for contributions, and, again, many less than 0.01.

While these results can be viewed as necessary for the stability of the coefficient vectors ξ_i , they are obviously not sufficient, for it could be that the estimates across the quarters are simply draws from populations of random numbers. To test for this contingency, we turn to tests on the ξ_i vectors in relation to one another. Specifically, two procedures have been pursued, the first a series of paired t-tests for a sample of 40 vector pairs chosen at random from the population of all possible pairs, and the second a sequence of homogeneous regressions (for the same sample of vector pairs) in which one vector in a pair is regressed on the other. A tally of the t-values that are two or greater (in absolute value) in the paired t-tests given in Table 3. The results for the tests themselves for both expenditures and budget shares are tabulated in Tables 4 and 5.

In Table 3, we find a total of 10 t-values for expenditures and 13 for budget shares that are greater than 2 in absolute value. This is only 2.2 percent of the 1040 vector-pairs in the sample, which is well under what would be expected from chance alone. However, of the 13 greater than 2 for budget shares, more than half (7) are in the coefficients for apparel. Apparel, accordingly, is the only category for which paired t-tests indicate stability of its ξ_i vectors to be questionable, but only in the coefficients for budget shares.

As a way of including all of the coefficients for all 40 quarters in a statistical test for instability, a paired t-test has been undertaken in which the coefficients for the expenditure categories are "stacked" and then "split" into two vectors of 20 quarters each, 1996Q4 through 2000Q4 and 2001Q1 through 2005Q4. The test, accordingly, is on differences in coefficients 20 quarters apart, 1996Q1-2001Q1, 1996Q2-2001Q2, and so on and so forth. The results for the expenditure and budget share coefficients are given in Table 6. Ignoring sign, the t-statistics are seen to be greater

Table 3

$$t = \frac{md}{sd} ,$$

where

$$sd = \frac{\left[\sum_{i=1}^{13} (d_i - md)^2\right]^{0.5}}{12}$$

Since a households remain in the quarterly surveys for up to five quarters, the quarters in the quarter-pairs in the sample are (with only a few exceptions) always at least five quarters in separation. A description of the sample is given in Appendix 1.

 $^{^{9}}$ For the formula that is used in the paired-tests, let d_{i} denote the difference between i^{th} coefficients in a vector-pair and md the mean of the difference. The t-statistic is then calculated as

T-Values in Paired T-Tests Greater Than 2 40 Vector-Pairs (Absolute Value)

Category	Expenditures*	Budget Shares**
Food	1	1
Alcoholic beverages	0	0
Housing	1	2
Apparel	0	7
Trans	1	0
Health	0	1
Entertainment	0	0
Personal Care	0	0
Reading	1	2
Education	0	0
Tobacco	2	0
Misc	2	0
Contributions	1	0
Personal Insurance	1	0

^{*} From Table 4.

than 2 for seven of the expenditure categories (Food, Alcoholic Beverages, Health, Personal Care, Reading, Contributions, and Personal Insurance) and for eight of the budget shares (Food, Alcoholic Beverages, Health, Entertainment, Personal Care, Education, Miscellaneous, and Contributions). While these large t-statistics appear to contradict the message in Tables 3-5 regarding stability of the ξ_i vectors, it should be kept in mind that, unlike for the paired t-tests on the 40 vector-pairs, for which the time distance between vectors is random, the time distance in Table 8 is a fixed 20 quarters. As will be seen in Section III, many of the coefficients in the categories with large t-statistics display significant trends.

The last exercise in this section involves the regression of one vector ξ_i in a pair on the other, i. e., the estimation of

(2)
$$\xi_{i} = \alpha + \beta \xi_{j} + \varepsilon , \quad i \neq j.$$

One can then straightforwardly test the hypotheses of stability through examination of the resulting estimates of α and β and the size of the R², specifically, in terms of the closeness of $\hat{\alpha}$ to 0 and $\hat{\beta}$ and the R² to 1. A summary of the results from estimating these regressions for the 40 vector-pairs for budget shares are tabulated in Tables 7 and 8. Table 7 gives means of the estimated intercepts, slope coefficients, and t-ratios, along with standard deviations of the intercepts and slope coefficients

^{**} From Table 5.

Table 4

Paired T-Tests for Sample of 40 Quarter-Pairs
14 CES Expenditure Categories
Intra-Budget Expenditure Coefficients
1996 Q1 - 2005 Q4

				Categories			
<u>Quarter-Pairs</u>	<u>Food</u>	<u>Alcbev</u>	<u>Housing</u>	<u>Apparel</u>	<u>Trans</u>	<u>Health</u>	<u>Entertn</u>
4	0.01	1 07	0.47	1 00	0.50	0.05	0.01
1	0.01	-1.07	-0.47	1.82	-0.58	-0.05	-0.21
2	-0.15	-1.39	0.08	0.61	-0.94	-1.23	-1.00
3	-0.17	-0.12	1.04	0.85	-0.95	0.68	-1.22
4	0.68	-1.33	-1.20	0.19	-0.57	1.04	-0.57
5	-1.29	-1.39	-1.12	0.12	-0.03	0.60	-0.06
6	0.76	0.96	-0.97	0.51	1.04	0.59	-2.32
7	0.01	1.47	0.57	-0.95	-0.87	-0.73	-0.98
8	-0.31	-0.56	0.60	-0.78	0.08	-1.11	-0.74
9	-0.10	-0.50	-1.79	0.39	-0.14	-0.88	0.07
10	-0.04	-1.12	0.73	0.30	-0.21	-1.02	-1.07
11	-1.27	-1.32	-1.64	1.25	-1.11	-0.83	0.43
12	-0.94	-1.21	-1.13	1.71	0.63	-0.39	-1.70
13	-0.10	-1.37	-1.21	0.95	-2.05	-1.63	-1.18
14	-0.85	-1.53	-0.87	0.87	-0.99	-1.07	-0.66
15	0.90	0.03	1.46	0.63	0.15	0.23	-0.16
16	-0.84	-0.06	0.04	-0.20	-0.46	0.03	1.02
17	1.29	-1.53	-0.59	0.03	-0.41	-1.39	-1.52
18	0.92	-1.52	-0.21	1.22	0.75	-0.83	-0.93
19	0.40	-0.12	-1.62	0.92	0.06	-0.85	-0.74
20	-1.43	-1.30	0.51	1.25	-0.18	0.59	0.95
21	0.64	-0.35	-0.37	1.13	0.53	-0.33	-1.33
22	2.02	-0.83	0.37	-1.57	-1.00	-1.19	-1.03
23	-0.68	-0.96	0.04	-0.60	0.09	-1.60	-1.24
24	-1.00	-1.36	-0.86	1.12	0.22	0.45	-1.11
25	-0.48	0.74	0.34	0.19	0.03	-0.93	0.52
26	0.13	0.73	-0.90	-0.48	-0.98	-1.62	-1.08
27	-0.69	-0.84	0.64	0.33	0.50	0.33	0.22
28	1.63	-0.36	0.29	-1.30	-0.12	-1.04	-1.01
29	1.84	-0.95	1.26	-0.24	0.95	0.37	0.25
30	0.64	-1.24	0.28	1.86	-1.57	1.10	0.10
31	0.94	-0.62	-0.42	-1.47	-0.45	-0.94	-1.11
32	0.93	0.30	0.96	1.72	0.53	1.30	-1.81
33	-0.54	-0.91	-1.01	1.37	-0.92	0.12	-1.35
34	-0.79	-1.42	0.89	0.26	-0.50	-1.03	-0.80
35	-0.30	-1.11	-1.46	-0.02	-0.38	-1.33	-1.37
36	-0.18	-1.55	-0.31	0.56	0.95	-0.77	0.83
37	0.76	-1.42	-0.36	1.46	-0.75	0.73	-1.34
38	-1.64	0.24	-1.54	1.73	-1.02	-0.55	-1.07
39	0.62	-1.14	-1.12	0.19	0.70	-0.91	-1.57
40	-0.55	-1.09	-2.05	1.28	-0.15	-0.63	-1.23

Table 4 cont'd

				Categories			
<u>Quarter-Pairs</u>	<u>Perscare</u>	Reading	Educ	<u>Tobacco</u>	Misc	<u>Contrib</u>	Ins
1	0.11	0.64	-0.74	1.35	1.35	-1.21	1.18
2	0.78	0.20	1.38	-1.48	-1.48	-1.63	-1.02
3	1.02	-0.11	-0.09	-0.59	-0.59	0.12	0.01
4	0.21	0.62	-0.13	-0.88	-0.88	-0.40	-0.72
5	-0.35	-0.42	0.45	0.55	0.55	-1.03	-1.01
6	-0.12	-1.09	-1.52	0.41	0.41	-1.44	0.68
7	-0.78	-1.31	-1.03	-0.38	-0.38	-0.43	-0.66
8	-1.02	-0.29	0.39	0.98	0.98	-1.92	0.07
9	0.21	0.53	0.50	1.27	1.27	-0.12	0.61
10	-0.83	-0.29	-0.14	-1.90	-1.90	-1.42	-1.44
11	0.67	1.03	-0.47	0.64	0.64	1.06	0.49
12	0.65	1.11	1.01	0.27	0.27	0.35	1.37
13	-0.54	0.03	0.62	0.99	0.99	-1.08	0.29
14	-0.92	0.81	0.87	-1.06	-1.06	-1.40	-1.53
15	-0.71	-0.42	-0.28	-0.88	-0.89	-0.32	0.39
16	0.71	1.33	-0.15	0.94	0.94	-0.87	-0.35
17	-0.73	-1.85	-1.02	-0.30	-0.30	-0.59	-0.23
18	-0.54	-0.26	0.72	-1.82	-1.82	-1.54	-0.94
19	0.59	0.65	0.08	-0.85	-0.85	-0.44	0.21
20	1.01	-0.89	0.67	0.99	0.99	-0.94	-0.99
21	-0.44	-1.38	-0.89	-0.90	-0.89	-0.07	-0.21
22	-0.69	-2.10	-0.39	-0.89	-0.89	-0.56	-1.08
23	-0.59	-1.05	1.22	-2.14	-2.14	-0.11	-1.46
24	-0.68	-0.66	-0.10	-2.03	-2.03	-1.32	-1.19
25	0.62	-0.06	-0.94	1.83	1.83	-0.76	-0.01
26	-0.56	-1.00	-1.43	-0.19	-0.19	-1.49	-0.96
27	-1.94	-1.18	0.14	0.75	0.75	0.85	-0.07
28	-0.96	-1.52	-0.27	0.07	0.07	-0.64	0.02
29	-1.96	0.04	-0.16	0.06	0.06	-1.45	0.18
30	-0.15	-0.26	-1.10	1.55	1.55	-1.61	0.31
31	-0.68	0.30	0.08	-1.58	-1.58	-0.98	-0.64
32	1.46	1.69	-0.81	-0.30	-0.30	-1.25	1.09
33	-0.36	0.73	-0.33	-1.39	-1.39	-2.52	-0.59
34	-0.63	-0.34	1.46	-1.78	-1.78	-1.15	-1.33
35	-0.38	0.35	-0.15	-1.36	-1.36	-1.32	0.78
36	0.27	1.02	0.84	0.14	0.14	0.83	0.98
37	-0.72	-0.96	-0.59	1.09	1.09	-0.16	1.18
38	1.68	-1.29	0.71	-0.34	-0.34	-0.42	-1.46
39	-0.74	-1.50	0.17	-1.07	-1.07	-0.74	-0.09
40	-0.57	-1.53	-1.07	0.13	0.13	-1.75	-1.37

Table 5

Paired T-Tests for Sample of 40 Quarter-Pairs
14 CES Expenditure Categories
Budget Share Coefficients
1996 Q1 - 2005 Q4

				Categories			
Quarter-Pairs	Food	<u>Alcbev</u>	<u> Housing</u>	<u>Apparel</u>	Trans	<u>Health</u>	Entertn
1	-0.07	1.22	-1.92	1.26	-0.86	-1.43	0.47
2	0.85	-1.45	-0.65	1.82	-1.43	-1.28	-1.19
3	1.32	-0.64	-0.65	-0.69	0.45	-0.48	-0.48
4	-0.41	-0.48	0.05	1.13	-0.74	0.13	0.02
5	0.15	-0.82	0.36	0.26	-0.92	0.56	-1.49
6	1.56	1.58	1.05	-0.19	1.03	0.37	0.63
7	1.85	-0.86	0.74	-1.55	1.25	-1.76	-0.99
8	1.96	-0.79	1.16	-1.68	1.05	-0.30	-2.12
9	-0.24	-0.44	-0.57	1.27	-0.85	0.11	-0.20
10	1.87	-1.24	1.16	-0.27	0.66	-2.04	-1.06
11	-1.78	0.75	-1.71	0.64	-1.41	-0.73	-0.37
12	-0.20	0.45	-0.00	2.12	-1.11	0.29	-0.74
13	0.50	-0.81	-1.93	1.11	-1.13	-1.34	-0.98
14	1.18	-1.95	-1.08	-1.21	-1.16	-1.45	-1.17
15	1.56	-1.93	0.45	0.38	0.59	-0.27	-1.02
16	-0.40	0.74	0.67	0.88	0.63	-0.68	-0.15
17	1.94	-1.12	-0.21	-1.66	0.41	-1.43	-1.21
18	1.45	-1.98	-2.15	1.27	-0.27	-1.69	-0.57
19	0.97	0.63	-1.03	0.69	-0.50	-0.03	-0.27
20	-0.82	-0.64	-0.48	2.60	-1.18	-0.10	-0.99
21	1.48	-0.47	0.43	3.47	-1.17	-0.89	-1.56
22	1.97	-1.39	0.80	-2.10	-0.32	-0.92	-1.04
23	0.84	-1.60	0.35	1.45	-0.27	-1.28	-0.87
24	1.43	-0.96	-0.71	-0.68	-1.38	-0.46	-1.29
25	0.46	0.79	0.72	-0.67	0.43	-1.65	-1.11
26	2.28	-0.17	1.14	-0.44	0.90	-1.37	-1.45
27	0.21	0.39	-0.50	-1.35	-1.66	-0.34	-1.38
28	1.89	-1.80	0.12	-2.00	0.71	-0.42	-1.77
29	1.96	-1.55	0.85	-0.01	1.92	0.35	-0.97
30	-1.48	-0.46	-1.01	1.59	-1.30	-1.53	0.66
31	1.80	-1.12	1.59	-2.32	-0.15	-0.27	-1.02
32	1.25	0.33	0.82	1.23	0.96	0.72	0.70
33	0.96	0.15	0.65	-0.45	-0.92	-1.60	-1.18
34	0.83	-1.79	-1.15	0.92	-0.98	-1.53	-0.91
35	1.65	-0.99	-0.97	3.74	-1.26	-1.33	-1.20
36	0.59	0.49	-0.00	-0.41	-0.31	0.72	-0.09
37	1.16	-0.43	-0.38	1.47	-1.71	-1.12	-1.34
38	-0.65	1.14	-1.33	1.40	-1.28	-1.20	-0.28
39	0.87	-1.30	-0.67	-1.22	-0.98	-1.11	-0.96
40	1.25	-0.31	-2.19	1.79	-1.30	-1.37	-1.25

Table 5 cont'd

				Categories			
Quarter-Pairs	Perscare	Reading	_Educ_	<u>Tobacco</u>	Misc	<u>Contrib</u>	Ins
1	0.82	0.65	1.56	1.28	0.27	-0.74	-0.82
2	-1.12	0.08	-0.23	0.07	-1.58	-1.54	-1.83
3	-1.30	0.72	0.11	0.85	-0.04	0.48	0.57
4	0.94	0.33	-1.34	-0.36	0.96	1.09	0.39
5	0.41	1.99	0.96	0.57	0.32	-1.11	0.67
6	-1.62	-1.05	-1.05	-0.95	0.48	-1.00	1.30
7	1.01	-0.58	-0.69	-0.16	0.33	-0.74	0.45
8	0.12	-0.88	-1.21	-0.54	0.08	-1.82	0.51
9	1.17	1.35	0.91	-0.69	1.20	-1.53	0.23
10	0.43	0.74	-0.84	-0.39	-0.81	-1.66	-0.22
11	1.48	0.56	-0.12	1.42	1.05	1.48	-0.15
12	-1.08	2.30	0.95	0.11	0.38	-0.24	0.13
13	0.98	0.78	-0.30	0.32	1.00	-1.43	-0.67
14	-0.15	-0.22	0.31	-1.48	-2.02	-1.55	-1.57
15	-1.12	-0.44	-1.66	-0.90	-1.51	-1.79	0.86
16	1.01	1.23	1.04	0.54	1.22	0.93	-0.21
17	-0.80	-1.41	-1.08	-0.60	0.48	-1.39	1.15
18	-0.65	-0.47	-0.81	-0.48	-1.19	-1.64	-0.81
19	-1.13	-0.41	-0.83	-0.05	-0.76	-0.48	0.65
20	0.70	0.93	-0.37	0.55	-0.37	0.31	0.98
21	-1.66	1.64	-1.95	-0.59	-0.91	-1.63	0.03
22	-0.55	-1.47	-0.94	-0.45	-0.31	-1.41	1.79
23	0.36	1.15	-0.76	0.32	-1.13	-1.44	-1.28
24	-0.98	0.06	0.06	-0.15	-1.46	-1.72	0.92
25	0.59	0.86	1.11	0.49	0.33	-0.51	-0.61
26	-1.30	0.93	-1.94	-1.18	0.02	-1.70	0.14
27	-0.57	-1.41	-0.39	-1.06	0.44	0.68	-0.27
28	-0.11	-1.43	-0.99	-0.49	-0.29	-1.53	1.42
29	-0.61	-0.97	-0.63	0.70	-0.88	-1.75	-0.74
30	1.08	0.71	1.44	0.32	1.42	0.34	-1.29
31	-0.89	-0.72	0.65	-0.41	-0.28	-1.56	1.12
32	-1.81	-0.20	0.41	0.23	-1.62	0.45	0.88
33	-1.35	-0.41	-1.52	-0.23	-1.52	-1.53	0.54
34	0.67	1.78	0.16	0.40	-1.51	-1.56	-1.62
35	-1.39	2.58	-1.36	-1.05	-0.04	-1.53	0.26
36	0.05	-1.34	-0.90	0.40	-0.52	0.16	1.00
37	-1.12	0.72	-0.25	0.21	0.50	-1.52	0.22
38	0.65	1.23	-1.87	-0.15	1.65	1.43	0.64
39	-1.17	-0.83	-0.43	-0.10	-1.02	-1.77	-0.28
40	-1.63	1.06	-0.43	-0.08	0.34	-1.52	-0.64

Table 6

Paired T-Tests For 20-Quarter Vector-Pairs
1996Q1 - 2000Q4 2001Q1 - 2005Q4

<u>Category</u>	<u>Expenditures</u>	Budget Shares
Food	2.38	6.18
Alcoholic beverages	-3.98	-4.33
Housing	-1.01	1.26
Apparel	0.80	1.16
Trans	0.55	-0.52
Health	-4.52	-4.63
Entertainment	-1.42	-5.30
Personal Care	-4.75	-3.79
Reading	-2.06	-0.79
Education	-1.23	-2.25
Tobacco	-0.03	-1.21
Misc	-1.67	-2.14
Contributions	-3.41	-7.26
Personal Insurance	-2.17	0.57

for the 40 regressions. Table 8 gives means and distribution statistics for the R²s of the equations. In Table 7, we see that with regard to the intercepts:

- (1). None of the mean t-ratios are in excess of 2 (in absolute value), and, with exception for transportation, all are less than 1;
- (2). With exception of reading, all of the means lie within a single standard deviation of 0;
- (3). For all 40 estimated equations, 52 of the 560 mean t-ratios (ignoring sign) are greater than 2, which is a little less than 10 percent. Of the 52, 31 are for apparel (6), transportation (10), entertainment (7), and reading (8). Zeroes are recorded for alcoholic beverages, education, and personal insurance.

For the slope coefficients, a key finding is that, except for contributions, all of the means are within a single standard deviation of 1. For the R^2 s in Table 8, we see that the mean R^2 for 8 of the 14 categories is greater than 0.80, with four (for apparel, health, entertainment, and tobacco) greater than 0.90. Food, housing, apparel, health, entertainment, and tobacco all have at least 25 of their 40 R^2 s greater than or equal to 0.90. At the other extreme, alcoholic beverages, transportation, education, and miscellaneous expenditures show the least stability by this measure, transportation with no R^2 s of 0.90 or more and education with just one.

Table 7

Summary Statistics
Estimation of Expression (2)
40 Vector-Pairs
Budget Shares

		Itercept				Slope		
			Mean	t-ratio			Mean	
<u>Category</u>	<u>Mean</u>	Std Dev	<u>t-ratio</u>	> 2	<u>Mean</u>	Std Dev	<u>t-ratio</u>	
Food	-0.0218	0.05	-0.36	5	1.30	0.32	10.96	
Alcoholic Beverages	0.011	0.004	0.09	0	0.83	0.25	6.95	
Housing	0.0327	0.08	0.42	2	0.95	0.19	13.08	
Apparel	0.0042	0.014	0.54	6	1.02	0.32	16.74	
Transportation	0.0606	0.068	1.25	10	0.60	0.41	2.53	
Health	-0.0102	0.10	0.14	5	0.88	0.58	23.40	
Entertainment	0.0044	0.02	0.58	7	0.84	0.22	18.99	
Personal Care	0.0002	0.002	-0.04	1	0.93	0.41	10.74	
Reading	0.006	0.003	0.73	8	1.02	0.53	10.16	
Education	0.0049	0.01	0.35	0	0.67	0.48	3.79	
Tobacco	1.1E-6	0.003	0.05	1	0.90	0.19	16.69	
Miscellaneous	0.0054	0.008	0.77	5	0.79	0.51	4.87	
Contributions	0.0036	0.004	0.71	2	0.44	0.37	11.96	
Insurance	0.0302	0.031	0.70	0	0.81	0.26	5.95	

Table 8 $R^2 s \ \, \text{From Estimation of Expression (2)} \\ 40 \ \, \text{Vector-Pairs} \\ \text{Budget Shares}$

Category	<u>Mean</u>	<u>Std Dev</u>	<u>Min</u>	<u>Max</u>	$\underline{R^2 \geq 0.90}$
Food	0.88	0.09	0.67	0.98	26
Alcoholic Beverages	0.73	0.17	0.23	0.97	7
Housing	0.89	0.11	0.52	0.98	25
Apparel	0.94	0.05	0.74	0.99	32
Transportation	0.34	0.25	0.01	0.84	0
Health	0.90	0.16	0.46	0.99	30
Entertainment	0.94	0.06	0.74	0.99	31
Personal Care	0.83	0.16	0.25	0.99	18
Reading	0.80	0.17	0.28	0.99	12
Education	0.47	0.29	0.01	0.93	1
Tobacco	0.92	0.08	0.59	0.99	29
Miscellaneous	0.56	0.26	0.01	0.96	4
Contributions	0.78	0.25	0.17	0.99	18
Insurance	0.63	0.25	0.15	0.98	9

III. STABILITY OF INDIVIDUAL COEFFICIENTS

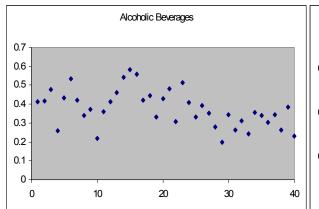
We now shift from investigation of the stability of the ξ_i vectors as single entities to an examination of the stability of individual coefficients. Visual displays of the 7280 estimated coefficients from the budget-share regressions are given in Figures 1 - 14. Each Figure includes charts showing estimates from the 40 surveys for each of the 13 intra-budget coefficients. In Figure 1, the first chart (labeled "Alcbev") shows the estimates from the 40 surveys for the coefficient on alcoholic beverages in the regressions of the budget shares for food on the 13 other budget shares, the second chart shows the same for the coefficients on housing, and so on and so forth. Readers are advised to view these charts in conjunction with Table 9, which categorizes the "scatters" in them under the headings, *Tight*, *Tight* (*Trend*), *Loose*, and *Loose* (*Trend*). Under the column headed "Food", for example, the coefficients on transportation and health are described as "tight", on housing and tobacco as "tight with a downward trend", etc. In Table 10, a tally is made from Table 9 showing the number of times that the estimated coefficient is classified as "tight", "tight with trend", etc.

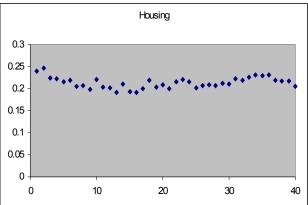
In approaching interpretation of the information in these Figures and Tables, it is important to keep in mind that present concern is not with making more precise the tests described earlier, but rather with examining stability of interdependency amongst budget shares. To this point, the focus has been on the macro-stability of the ξ_i vectors across surveys. In Figures 1 - 14 and Tables 9 and 10, the focus is on the elements of the ξ_i across both expenditure categories and surveys. In Table 8, the estimated coefficients on "food", for example, have been characterized as "tight" (as depicted in the graphs in Figures 2 - 14) in the budget share regressions for transportation, health, and personal insurance, and as "tight with trend" in the regressions for housing, apparel, personal care, and tobacco. Similarly, the estimated coefficients on "health" (as depicted in Figures 1 - 5 and 7 -14) are characterized as "tight" in the regressions for food and miscellaneous expenditures, and as "loose with trend" in the regressions for entertainment, personal care, and contributions. Noteworthy examples of "tightness" are evident in Figure 1 in the graphs for housing and transportation (i.e. in the coefficients on housing and transportation in the regressions with budget shares for food as the dependent variable), in Figure 3 for housing in the graphs for food and insurance, in Figure 5 for transportation in the graphs for food, housing, and insurance, and in Figure 14 for insurance in the graphs for housing and transportation. However, even cursory examination of the graphs makes clear that connections of greatest stability are between food and housing.

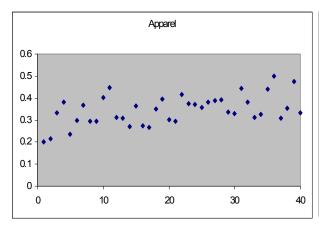
In Table 10, we see that the coefficients in ξ_i vectors displaying greatest stability are for food and housing, followed by apparel, transportation, health, and entertainment. In contrast, the coefficients with least stability are obviously those in the vectors for reading, education, and contributions, with tobacco and miscellaneous not far behind. The coefficients on food and housing are especially interesting, not just for their minimal variation, but also for their small, but perceptible, off-setting trends, upward in the coefficients on housing in the equations for the budget shares for food and downward on food in the equations for housing. Other stable coefficients are for health (no trend), transportation and insurance (both with upward trends), and personal care and tobacco (both with downward trends) in the ξ_i vector for food, and for transportation (no trend) and entertainment and insurance (both with upward trends) in the vector for housing.

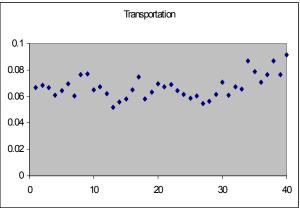
¹⁰ Classification has been informal by eye, and some readers, upon examining the charts at length, may reach different conclusions.

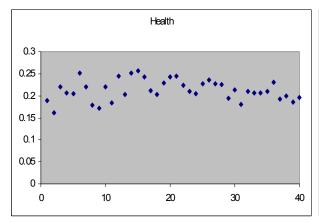
Figure 1
Estimated Budget-Share Coefficients
Food
1996Q1 - 2005Q4











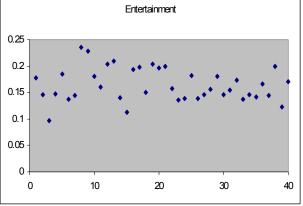


Figure 1 cont'd

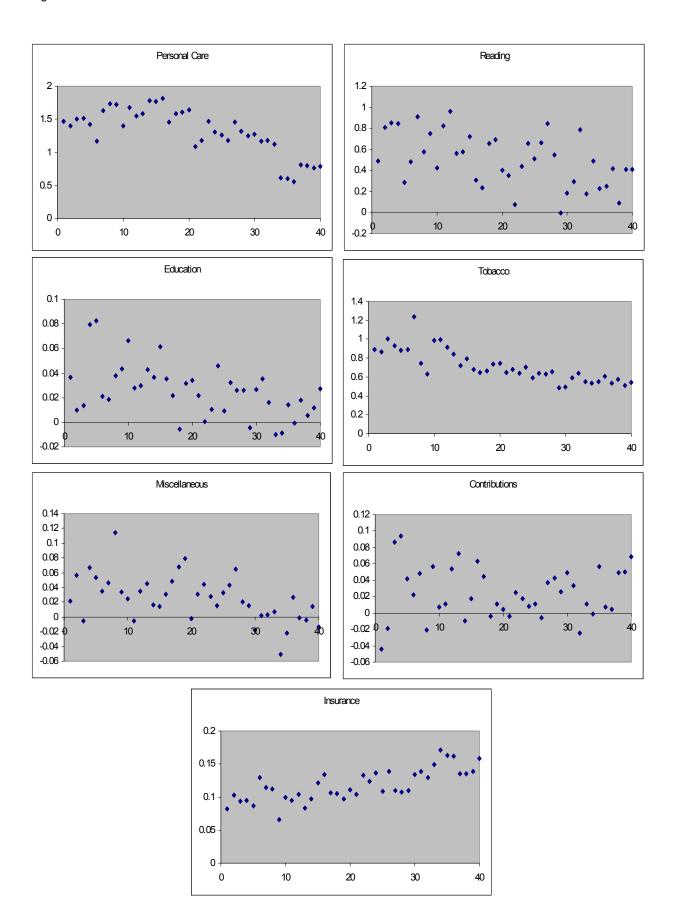
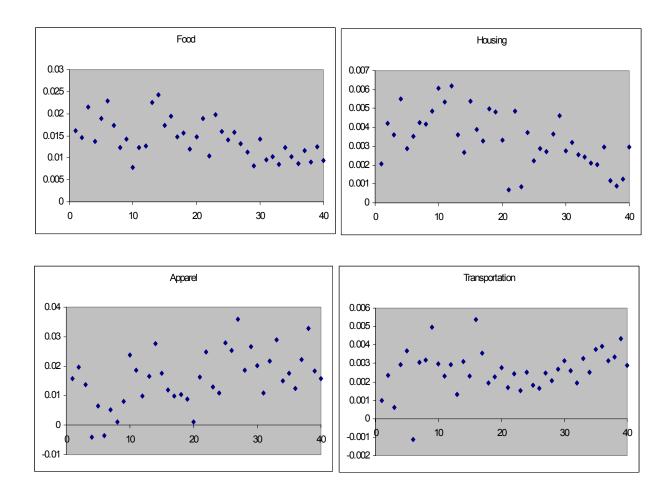
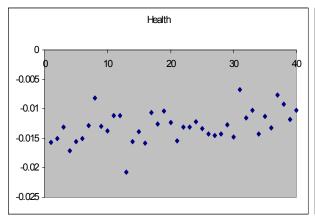


Figure 2
Estimated Budget-Share Coefficients
Alcoholic Beverages
1996Q1 - 2005Q4





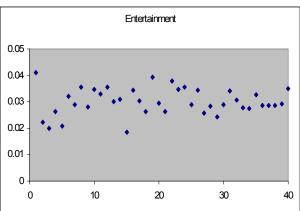


Figure 2 cont'd

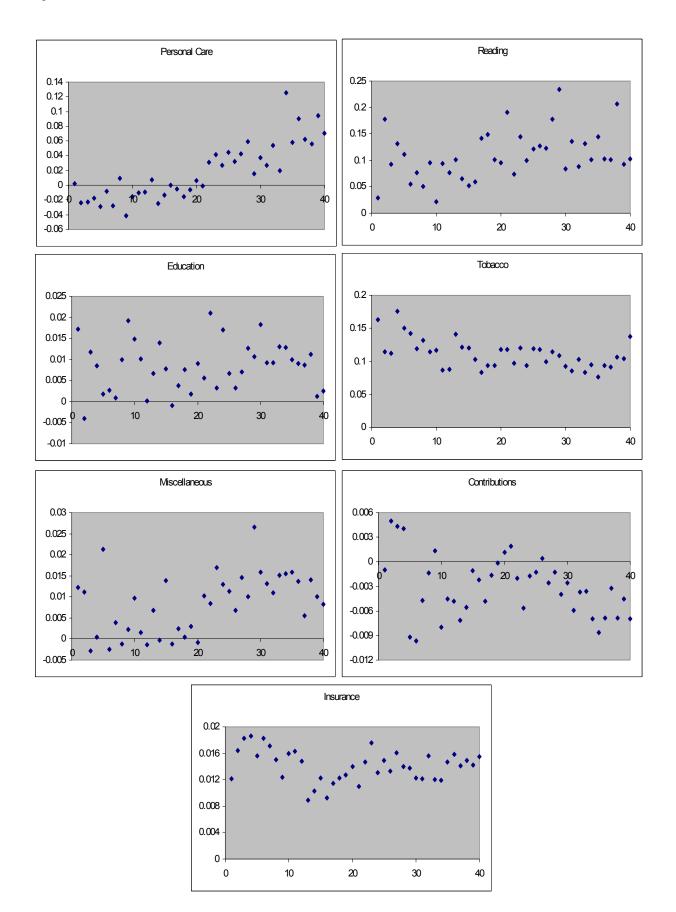
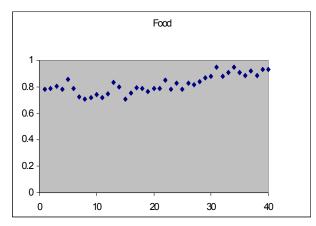
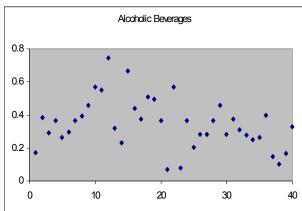


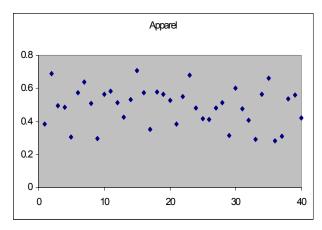
Figure 3

Estimated Budget-Share Coefficients

Housing
1996Q1 - 2005Q4







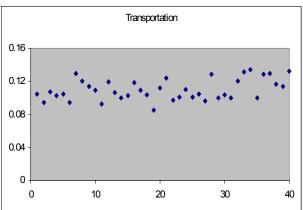
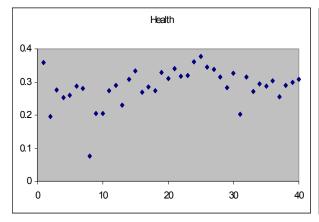
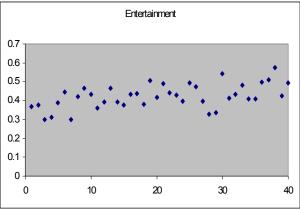


Figure 3 cont'd





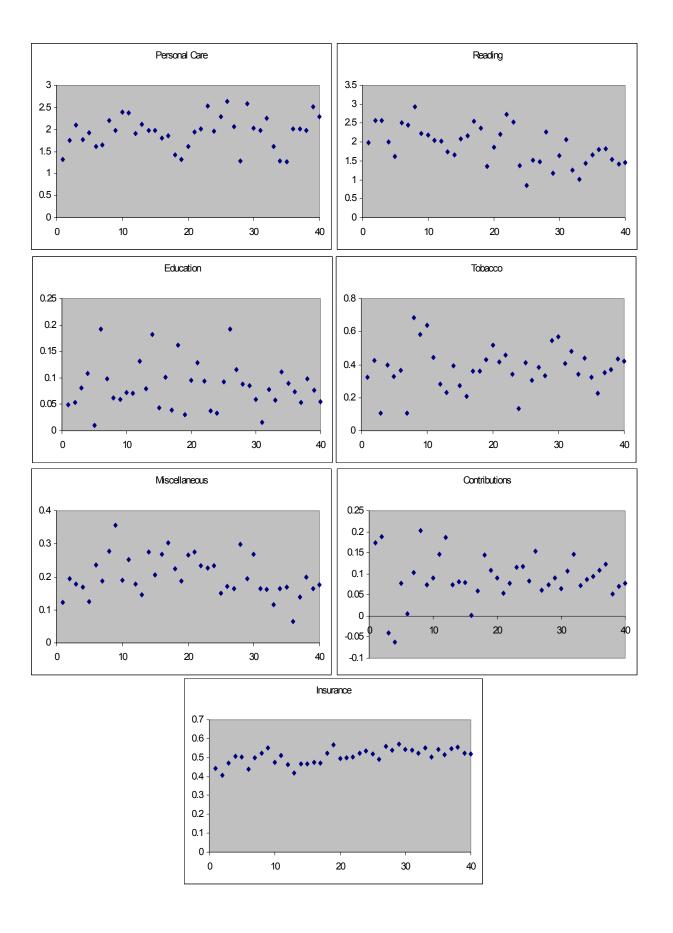
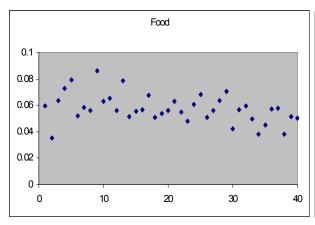
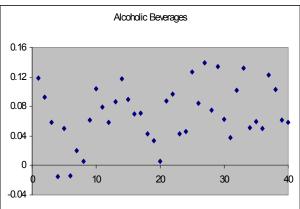


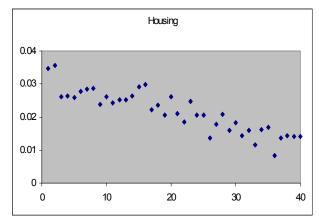
Figure 4

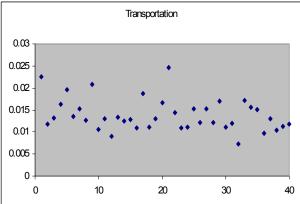
Estimated Budget-Share Coefficients Apparel 1996Q1 - 2005Q4

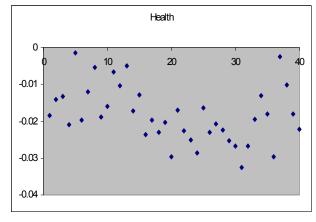
Figure 4 cont'd

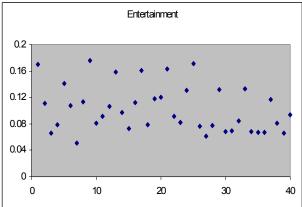












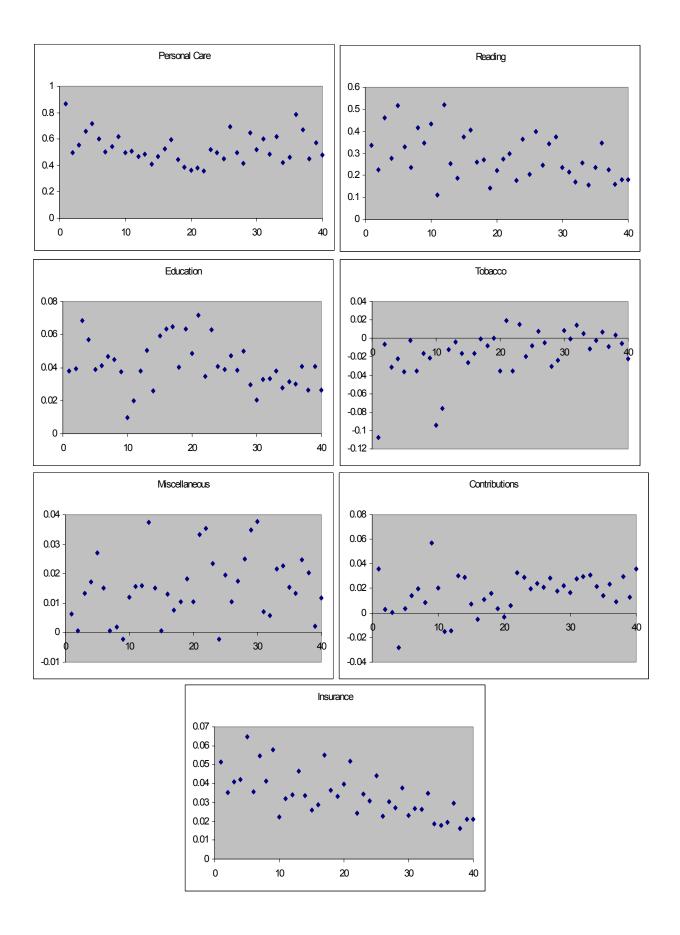
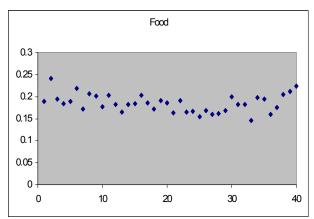
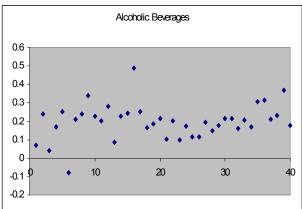
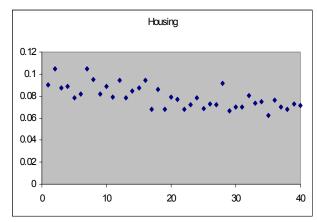


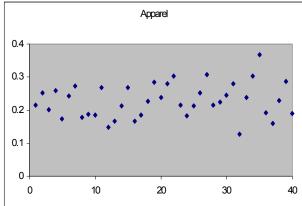
Figure 5

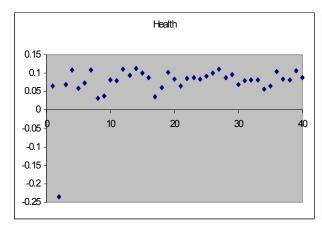
Estimated Budget-Share Coefficients
Transportation
1996Q1 - 2005Q4











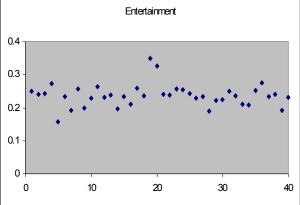


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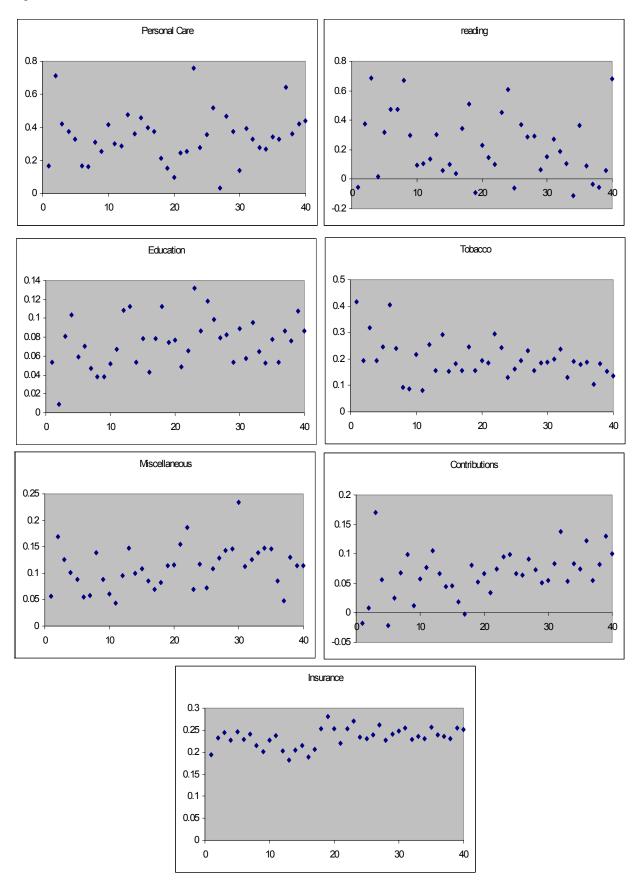
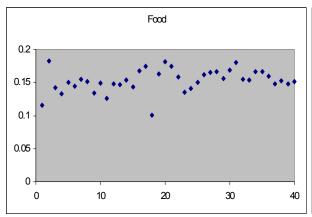
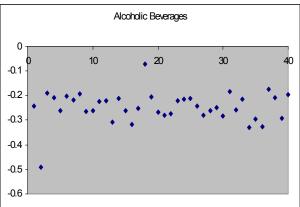
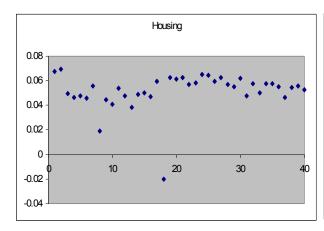


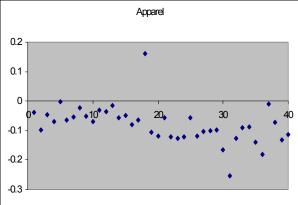
Figure 6

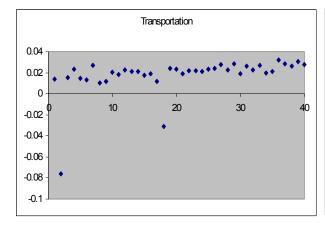
Estimated Budget-Share Coefficients
Health
1996Q1 - 2005Q4











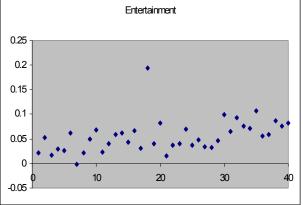


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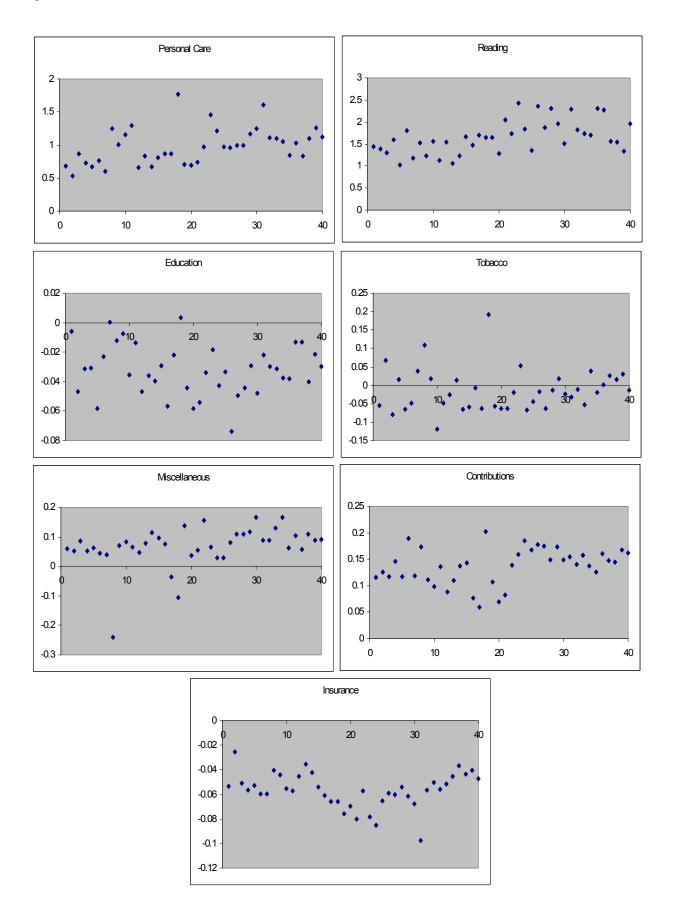
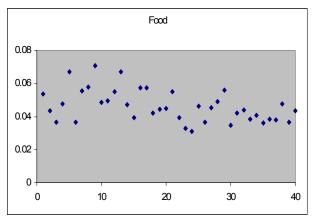
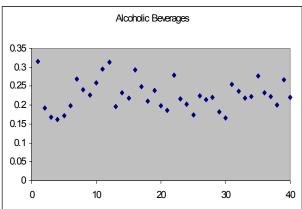
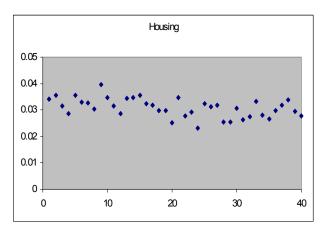


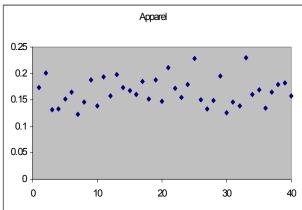
Figure 7

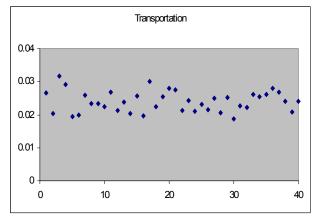
Estimated Budget-Share Coefficients
Entertainment
1996Q1 - 2005Q4











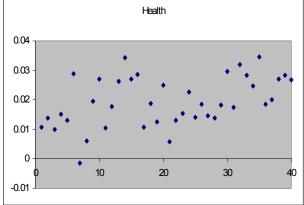


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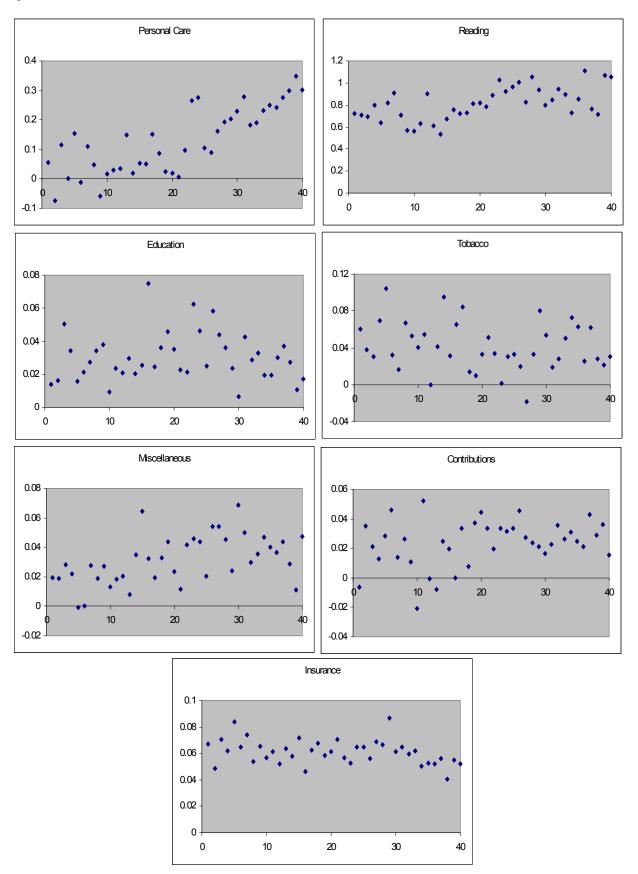
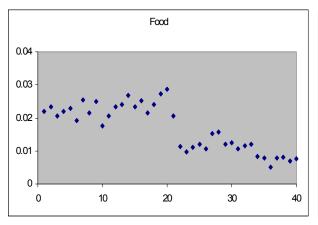
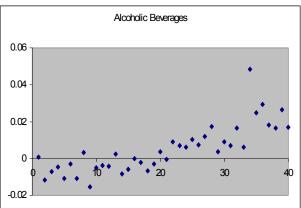
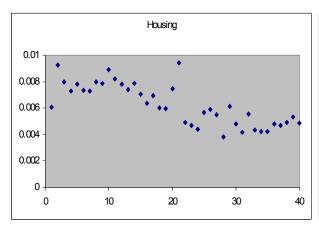


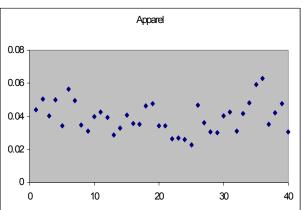
Figure 8

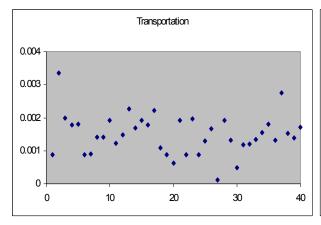
Estimated Budget-Share Coefficients
Personal Care
1996Q1 - 2005Q4











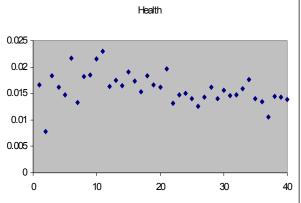


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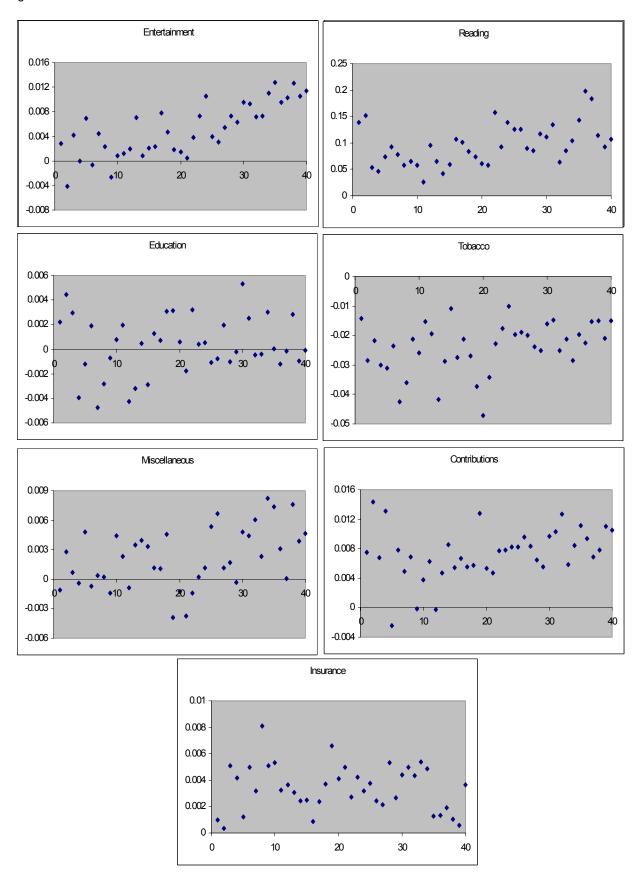
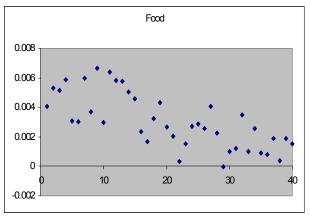
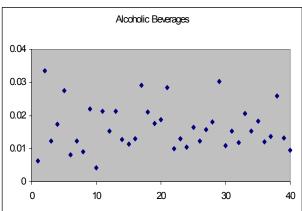
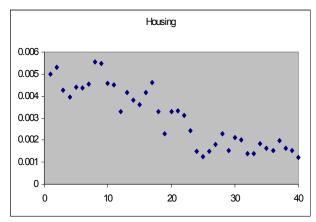


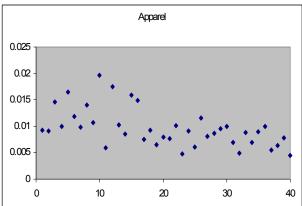
Figure 9

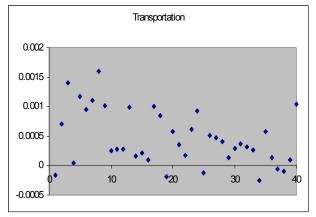
Estimated Budget-Share Coefficients
Reading
1996Q1 - 2005Q4











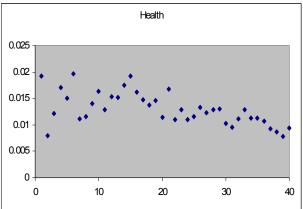


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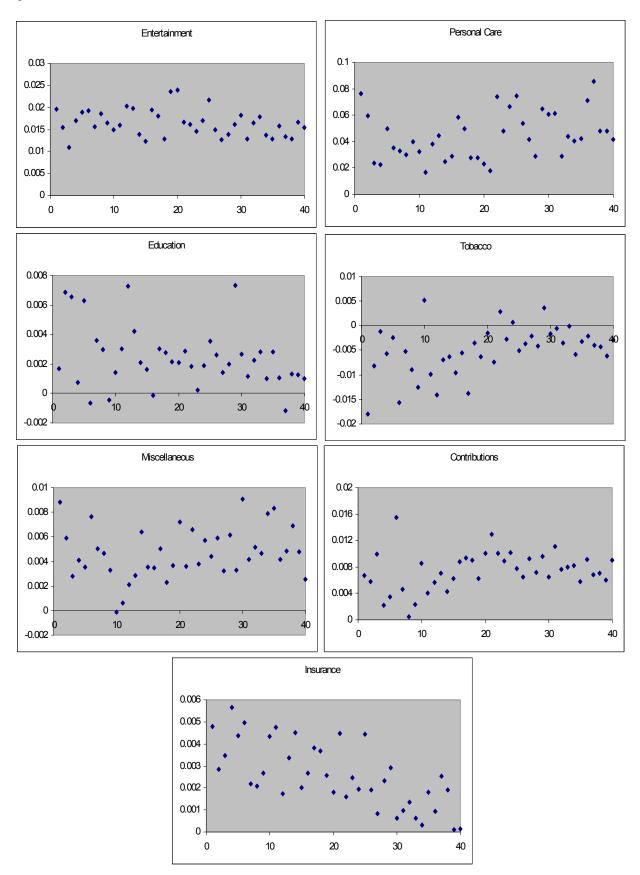
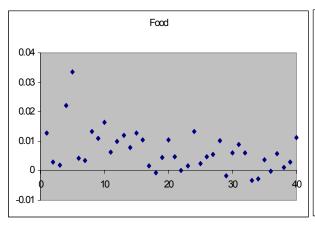
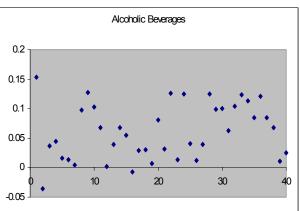
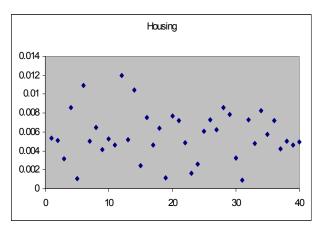


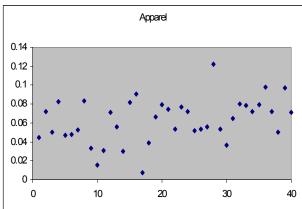
Figure 10

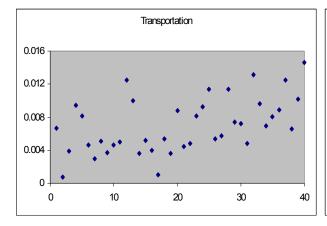
Estimated Budget-Share Coefficients
Education
1996Q1 - 2005Q4











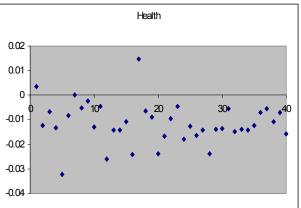


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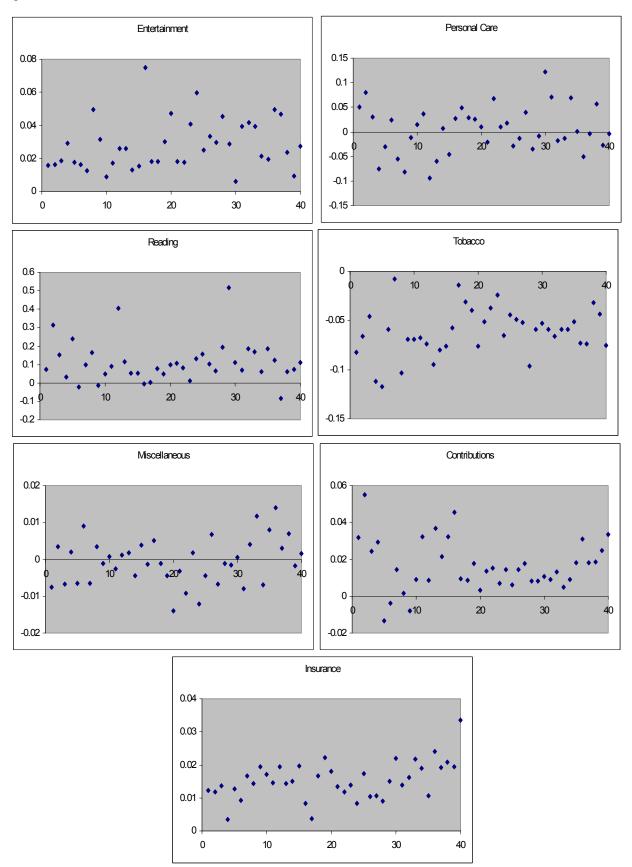
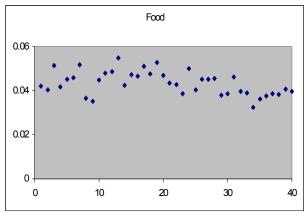
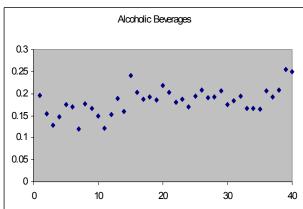
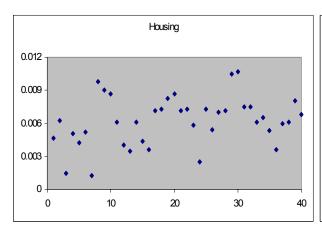
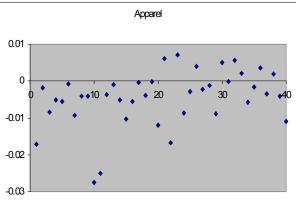


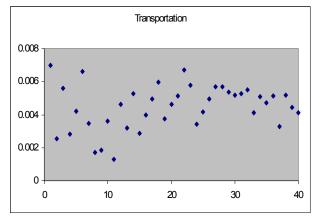
Figure 11
Estimated Budget-Share Coefficients
Tobacco
1996Q1 - 2005Q4











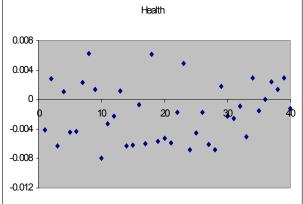


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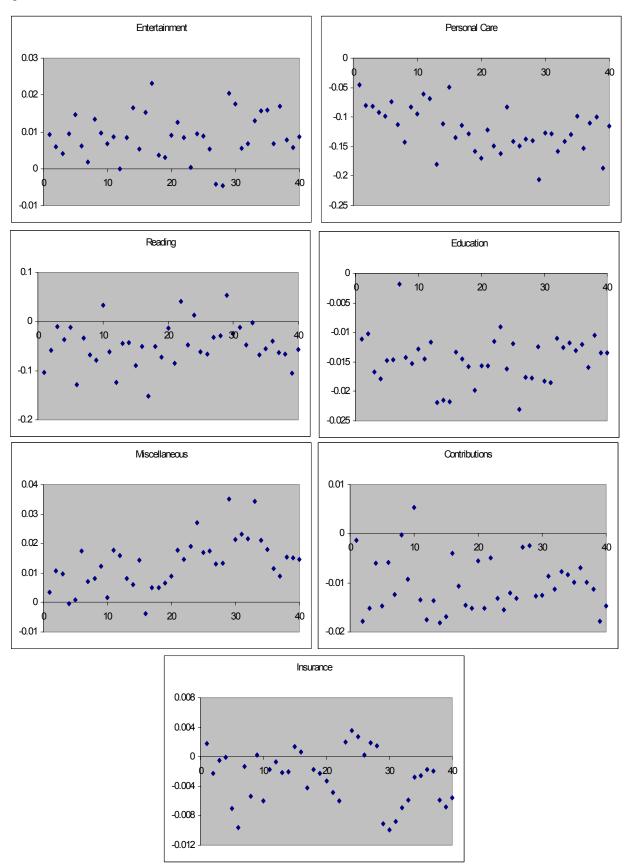
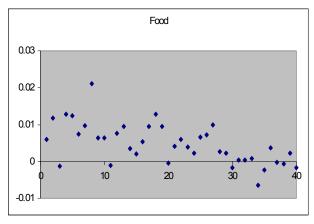
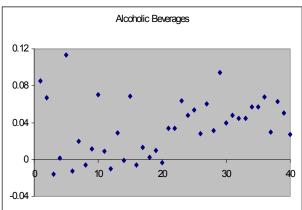
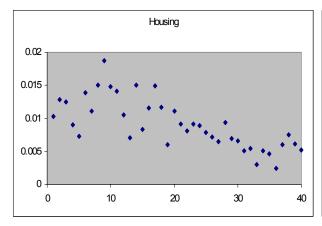
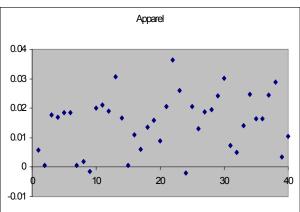


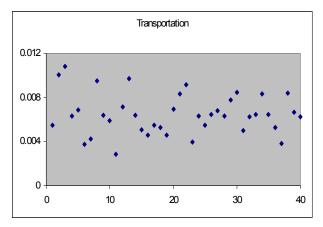
Figure 12
Estimated Budget-Share Coefficients
Miscellaneous
1996Q1 - 2005Q4











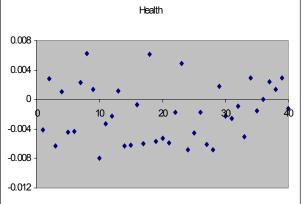


Figure 12 cont'd

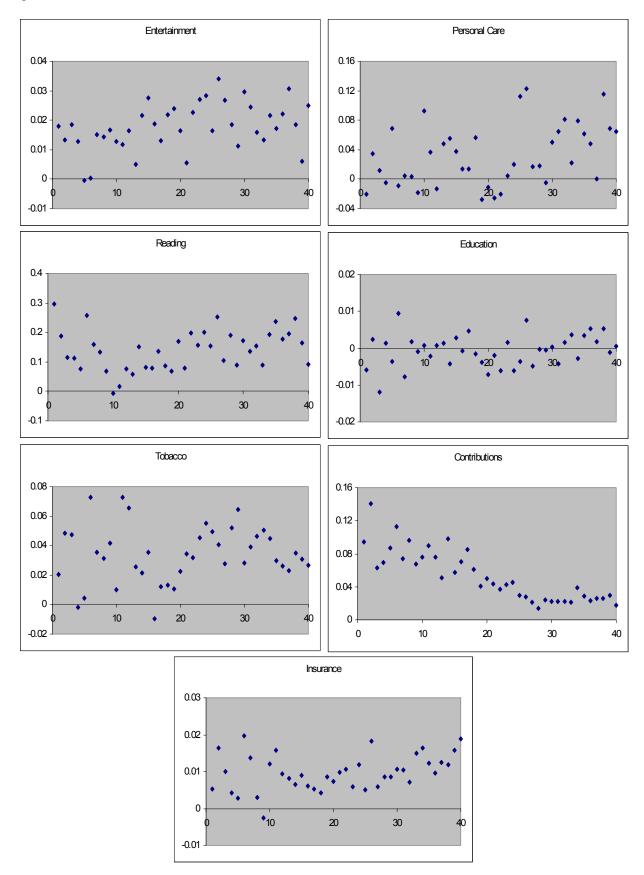
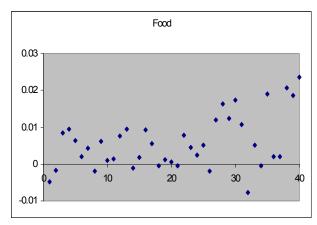
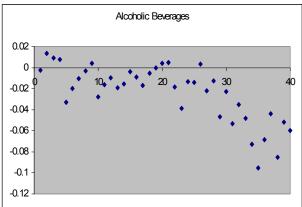
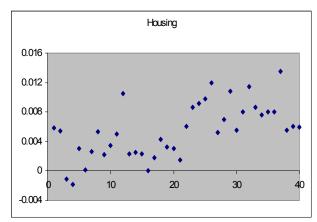


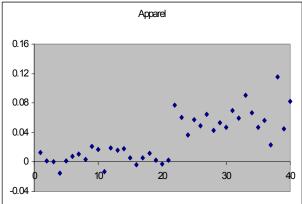
Figure 13

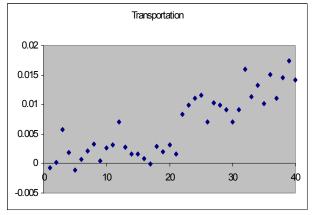
Estimated Budget-Share Coefficients
Contributions
1996Q1 - 2005Q4











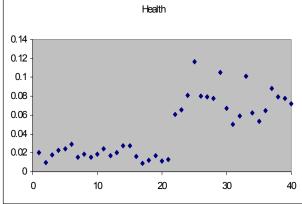


Figure 13 cont'd

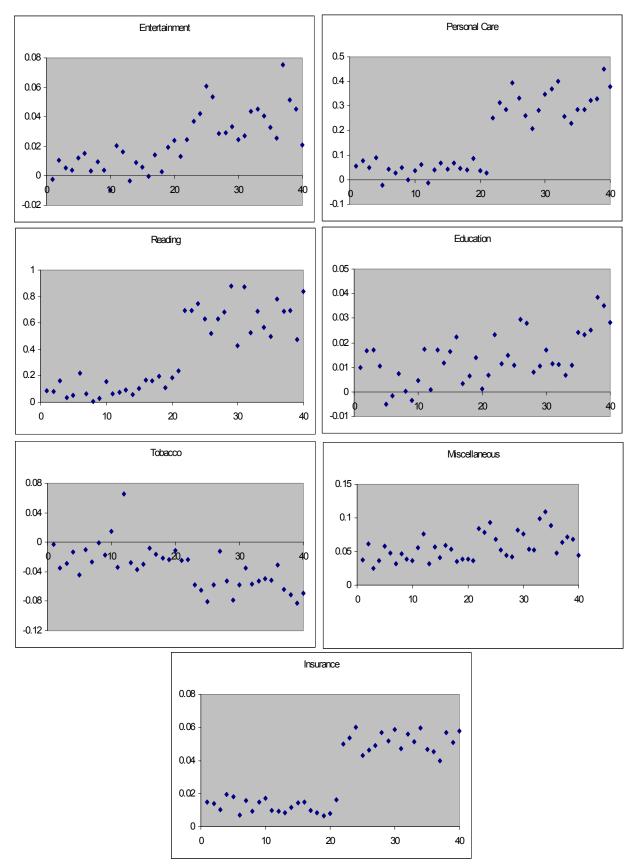
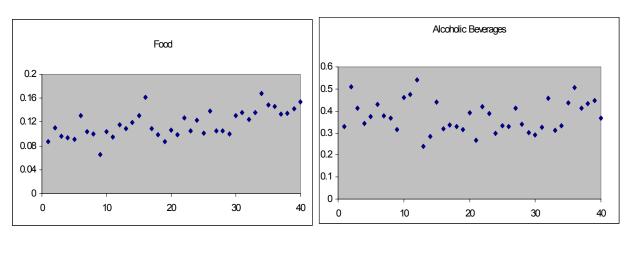


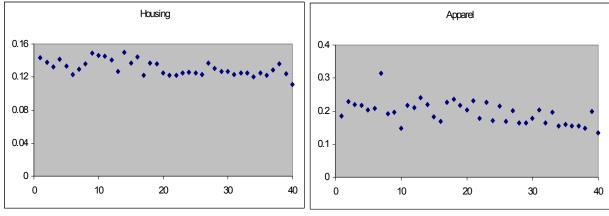
Figure 14

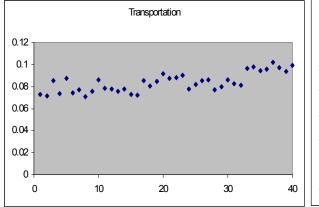
Estimated Budget-Share Coefficients

Personal Insurance

1996Q1 - 2005Q4







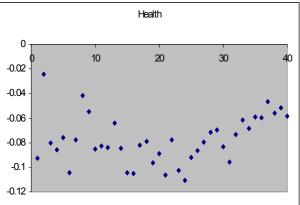


Figure 14 cont'd

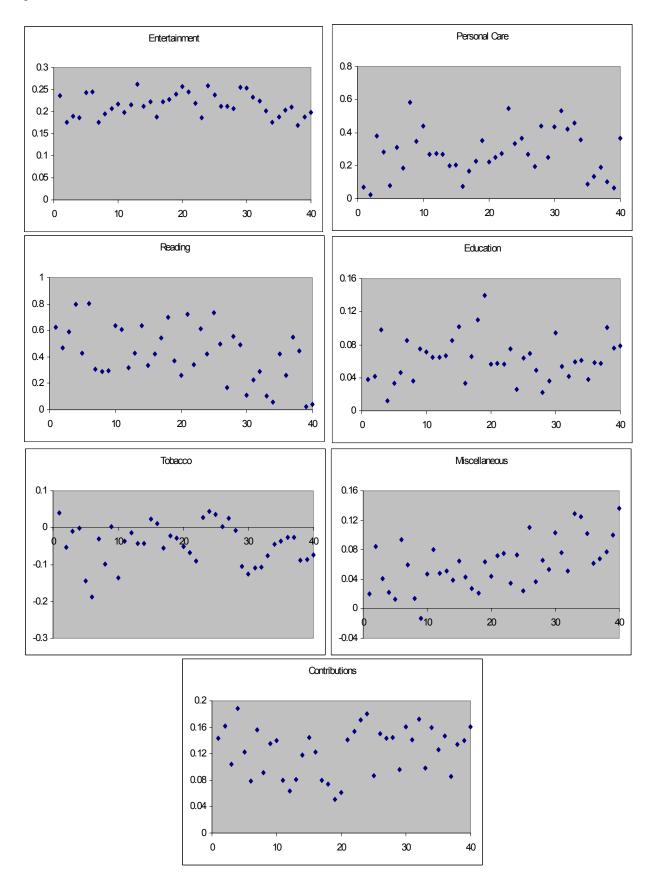


Table 9

Summary of Budget Share Coefficient Estimates
CES 14 Categories of Expenditure
1996 Q1 - 2005 Q4
(Figures 1 - 14)

			Exper	nditure Catego	ry		
<u>Relationship</u>	<u>Food</u>	<u>Alcbev</u>	<u>Housing</u>	<u>Apparel</u>	Trans	<u>Health</u>	<u>Entertn</u>
Tight	Trans Health	Entertn Ins		Perscare	Food Health Entertn	Food Apparel	Alcbev Trans
Tight(trend)	Apparel(U) Housing(D) Tobac(D) Perscare(D) Ins(U)	Health(U) Perscare(U) Tobac(D)	Food(U) Trans(D) Health(U) Entertn(U) Perscare(U) Ins(U)	Food(D) Housing(D)	Housing(D) Ins(U)	Alcbev(D) Housing(U) Entertn(U) Misc(U)	Reading(U) Housing(D)
Loose	Contrib	Trans Reading Educ Misc	Apparel Educ Tobac Contrib	Alcbev Health Entertn Tobac Misc Contrib	Alcbev Apparel Perscare Educ Misc	Educ Tobac Ins	Apparel Educ Tobac Contrib
Loose(trend)	Alcbev(D) Reading(D) Entertn(D) Educ(D) Misc(D)	Food(D) Housing(D) Apparel(U) Contrib(D)	Alcbev(D) Reading(D) Misc(D)	Trans(D) Reading(D) Ins(U)	Reading(D) Tobac(D) Reading(U) Contrib(U)	Perscare(U) Contrib(U)	Food(D) Health(U) Perscare(U) Misc(U) Ins(D)
			Exper	nditure Catego	ry		
Relationship	<u>Perscare</u>	<u>Reading</u>	<u>Educ</u>	<u>Tobacco</u>	<u>Misc</u>	<u>Contrib</u>	<u>Ins</u>
Tight	Apparel				Health		Alcbev Entertn
Tight(trend)	Food(D) Alcbev(U) Housing(D)	Housing(D) Health(D)		Food(D) Alcbev(U)	Contrib(D)	Apparel(U) Misc(U)	Food(U) Housing(D) Appare1(D) Trans(U)
Loose	Trans Educ Contrib Ins	Alcbev Entertn Perscare Educ Misc Contrib	Alcbev Housing Apparel Health Entertn Perscare Reading Tobac Misc	Housing Apparel Trans Health Entertn Reading Educ Contrib Ins	Alcbev Apparel Trans Perscare Reading Educ Tobac Ins		Health Perscare Educ Tobac Contrib
Loose(trend)	Health(D) Entertn(U) Reading(U) Tobac(U) Misc(U)	Food(D) Apparel(D) Trans(D) Tobac(U) Ins(D)	Food(D) Trans(U) Contrib(D) Ins(U)	Perscare(D) Misc(U)	Food(D) Housing(D) Entertn(U)	Food(U) Alcbev(D) Housing(U) Trans(U) Health(U) Entertn(U)	Reading(D) Misc(U)

Perscare(U)
Reading(U)
Educ(U)
Tobac(D)
Ins(U)

Note: (U) Upward Trend; (D) Downward Trend.

Table 10

Distributions of Budget Share Coefficients (From Table 11)

<u>Category</u>	<u>Tight</u>	<u>Tight(trend)</u>	Loose	<u>Loose(trend)</u>
Food	2	5	0	6
Alcoholic Beverages	2	2	6	3
Housing	1	7	2	3
Apparel	2	3	5	3
Transportation	3	2	4	4
Health	3	3	4	3
Entertainment	3	2	5	3
Personal Care	1	3	5	4
Reading	0	1	4	8
Education	0	0	10	3
Tobacco	0	2	7	4
Miscellaneous	0	2	5	6
Contributions	0	1	8	4
Insurance	1	3	4	5

The reciprocal relationships for food and housing are sufficiently tight over the 40 surveys not to wonder whether something may be involved that is beyond mere short-term historical association. However, before speculating on what this might be, a summary of statistical results from the analyses to this point is in order:¹¹

- (1). Regression equations in which the ξ_i vectors for each of the 14 categories of expenditure are "stacked" and then regressed on quarter dummy variables show no evidence of "level" effects amongst the 40 quarters of surveys for either expenditures or budget shares.
- (2). Paired t-tests of the hypothesis that there is no difference between individual coefficients of different ξ_i vectors undertaken on a sample of 40 survey pairs show that, of the 520 t-ratios calculated, only 10 are greater than 2 (in absolute value) for expenditures and 13 for budget shares. Of the 13 for budget shares, 7 are for the coefficients on tobacco.
- (3). Similar paired t-tests undertaken on bifurcation of the 40 surveys into

 $^{^{\}rm 11}\,$ Economic interpretations will be given in Sections IV and V.

1996Q1-2000Q4 and 2001Q1-2005Q4, in which ξ_i vectors separated by a constant 20 quarters in time are paired, show 7 of 13 t-ratios greater than 2 for expenditures and 8 of 13 for budget shares. This large jump in the rejection of the hypothesis of "no difference" is due to trends in coefficients, as recorded in Table 9 of this section.¹²

- (4). The most stringent test of stability in ξ_i vectors is through regressions in which one vector in a pair of ξ_i vectors is regressed on the other, as in expression (2) above. Stability is then judged in terms of the closeness of the intercepts to 0 and slope coefficients to 1. These regressions have been estimated for budget shares for ξ_i s in the sample of 40 vector-pairs, with results (as summarized in Table 7) consistent with stability.
- (5). However, stable ξ_i s with small variance are obviously not universal, as is evident in the small and highly variable R^2 s in Table 8 (for the sample of 40 vector-pairs) for transportation, education, and miscellaneous expenditures.

Results to this point thus suggest a reasonable stability in the ξ_i vectors for food, housing, apparel, health, entertainment, and tobacco, as well as in certain of their elements for alcoholic beverages, personal care, and insurance. The next task is to analyze the "strengths" and "forces" that may be represented in these relationships, which in this context can be assumed to consist of two properties, *size*, as reflected in numerical values (including signs) of the estimated regression coefficients, and *statistical strength*, as reflected in the estimates' t-ratios.¹³ Evident quantities to use as these parameters are the mean values of the estimated coefficients and associated t-ratios over the 40 surveys. These are tabulated in Tables 11-13, coefficient means for expenditures and budget shares in Tables 11 and 12, and mean t-ratios for budget shares in Table 13. (Mean t-ratios for the estimated expenditure coefficients are very similar to those for budget shares, and are consequently not tabulated.)

At this point, it becomes useful to define a second class of vectors, in this case *row vectors*, whose elements are the coefficients attaching to individual expenditure categories. These row vectors are of interest, especially to the sequel, because they can be interpreted as representing the "effects"

¹² Cf., Tables 6 and 9.

Numerical values measure the quantitative impacts that expenditures in different categories have on one another, while *signs* are instrumental in determining whether the impacts are positive (whether because two categories both have positive income elasticities of demand or simply because they are complements in consumption) or negative (because of being substitutes). *Statistical strength* can be seen as an indicator of the basic strength of impact ("clarity of signal", "insistence on being heard", etc.) regardless of magnitude.

that individual categories, as categories, may have on their sister categories of spending. If, for example, if the coefficients on food (which will indeed be found to be the case) are stable across surveys, one may be able to postulate a "food" effect in consumption that is an additional feature of tastes and preferences. For notation, let these vectors be denoted by Ψ_j , for j=1,...,14. The vector for food, Ψ_1 , then, will have -1 as its first component, the coefficient for food in the regression for alcoholic beverages as its second component, and so on and so forth. In Tables 11 and 12, accordingly, the Ψ_j vectors are represented by the coefficients in rows, rather than in columns as is the case for the ξ_i vectors. In Table 13, we find many t-ratios with large mean values in the Ψ_j vectors for food, housing, and personal insurance, which presages force and strength. Stability, though, awaits further investigation.

Table 11

Means of Intra-Budget Regression Coefficients
14 Categories of U.S. Consumption Expenditure
1996 Q1 - 2005 Q4

				Categories			
<u>Coefficients</u>	<u>Food</u>	<u>Alcbev</u>	<u> Housing</u>	<u>Apparel</u>	Trans	<u>Health</u>	<u>Entertn</u>
Food	-1.0000	0.0251	0.5561	0.0810	0.3378	0.0662	0.1058
Alcbev	0.5526	-1.0000	1.0240	0.1618	0.4045	-0.1798	0.4259
Housing	0.0692	0.0058	-1.0000	0.0422	0.1809	0.0163	0.0869
Apparel	0.2318	0.0165	0.5669	-1.0000	0.2153	0.0001	0.1358
Trans	0.0123	0.0006	0.0279	0.0038	-1.0000	0.0034	0.0153
Health	0.0741	-0.0092	0.0852	0.0145	0.1080	-1.0000	0.0319
Entertn	0.0480	0.0079	0.1681	0.0322	0.1500	0.0139	-1.0000
Perscare	1.0484	0.0584	2.7116	1.0965	1.2542	0.6811	0.2417
Reading	0.6835	0.1430	2.9127	0.5761	0.0970	1.2433	1.0551
Educ	0.0536	0.0011	0.1539	0.0170	0.0526	0.0055	0.0015
Tobacco	0.3067	0.1208	-0.4507	0.0027	0.4380	-0.0331	0.0457
Misc	0.0180	0.0046	0.2252	0.0303	0.0570	0.0471	0.0167
Contrib	0.0190	0.0014	0.1137	0.0111	0.0464	0.0346	0.0242
Ins	0.0856	0.0083	0.4041	0.0178	0.2077	0.0007	0.0529

Table 11 cont'd

				Categories			
<u>Coefficients</u>	<u>Perscare</u>	Reading	Educ	<u>Tobacco</u>	Misc	<u>Contrib</u>	<u>Ins</u>
Food	0.0120	0.0045	0.0641	0.0137	0.0047	0.0322	0.2321
Alcbev	0.0139	0.0214	0.0307	0.1220	0.0746	0.0836	0.4942
Housing	0.0132	0.0061	0.0317	-0.0021	0.0178	0.0288	0.1432
Apparel	0.0190	0.0062	0.0452	-0.0004	0.0264	0.0747	0.1305
Trans	0.0005	0.0000	0.0020	0.0008	0.0018	0.0016	0.0205
Health	0.0082	0.0089	0.0068	-0.0016	0.0393	0.0519	0.0022
Entertn	0.0011	0.0034	0.0022	0.0013	0.0010	0.0183	0.0683
Perscare	-1.0000	0.0479	0.1334	-0.1040	0.0499	0.2091	1.1773
Reading	0.0851	-1.0000	0.3017	-0.0568	0.1459	0.6886	1.9317
Educ	0.0016	0.0019	-1.0000	-0.0055	0.0069	0.0330	0.0643
Tobacco	-0.0262	-0.0087	-0.1413	-1.0000	0.0650	-0.1100	-0.1571
Misc	0.0010	0.0015	0.0093	0.0039	-1.0000	0.0727	0.0024
Contrib	0.0013	0.0024	0.0394	-0.0034	0.0460	-1.0000	0.0735

Ins 0.0049 0.0048 0.0272 -0.0026 -0.0064 0.0247 -1.0000

Table 12

Means of Budget-Share Coefficients

14 Categories of U.S. Consumption Expenditure

1996 Q1 - 2005 Q4

				Categories			
<u>Coefficients</u>	Food	<u>Alcbev</u>	<u>Housing</u>	<u>Apparel</u>	Trans	<u>Health</u>	<u>Entertn</u>
Food	-1.0000	0.0142	0.8194	0.0575	0.1850	0.1531	0.0463
Alcbev	0.3773	-1.0000	0.3455	0.0703	0.1985	-0.2466	0.2266
Housing	0.2135	0.0034	-1.0000	0.0217	0.0795	0.0519	0.0309
Apparel	0.3460	0.0158	0.4908	-1.0000	0.2285	-0.0797	0.1658
Trans	0.0671	0.0026	0.1100	0.0139	-1.0000	0.0179	0.0240
Health	0.2127	-0.0129	0.2860	-0.0182	0.0743	-1.0000	0.0194
Entertn	0.1646	0.0301	0.4235	0.1027	0.2367	0.0554	-1.0000
Perscare	1.3151	0.0186	1.9398	0.5315	0.3413	0.9789	0.1301
Reading	0.5037	0.1089	1.9048	0.2843	0.2267	1.6586	0.8123
Educ	0.0257	0.0084	0.0839	0.0414	0.0742	-0.0324	0.0302
Tobacco	0.7145	0.1111	0.3787	-0.0166	0.1986	-0.0137	0.0414
Misc	0.0253	0.0084	0.2017	0.0154	0.1106	0.0682	0.0313
Contrib	0.0249	-0.0032	0.0904	0.0161	0.0660	0.1373	0.0237
Ins	0.1182	0.0141	0.5067	0.0342	0.2333	-0.0566	0.0611

Table 12 cont'd

	Categories							
Coefficients	<u>Perscare</u>	Reading	<u>Educ</u>	<u>Tobacco</u>	_Misc_	<u>Contrib</u>	Ins	
Food	0.0170	0.0031	0.0070	0.0435	0.0047	0.0059	0.11684	
Alcbev	0.0049	0.0164	0.0610	0.1840	0.0356	-0.0237	0.37546	
Housing	0.0063	0.0030	0.0056	0.0062	0.0091	0.0055	0.13096	
Apparel	0.0393	0.0096	0.0624	-0.0049	0.0150	0.0317	0.19486	
Trans	0.0015	0.0005	0.0070	0.0044	0.0065	0.0065	0.08375	
Health	0.0159	0.0130	-0.0115	-0.0017	0.0166	0.0457	-0.07760	
Entertn	0.0051	0.0164	0.0281	0.0091	0.0178	0.0229	0.21487	
Perscare	-1.0000	0.0446	0.0040	-0.1207	0.0319	0.1726	0.27500	
Reading	0.0964	-1.0000	0.1115	-0.0501	0.1402	0.3718	0.42304	
Educ	0.0003	0.0024	-1.0000	-0.0147	-0.0007	0.0135	0.06243	
Tobacco	-0.0239	-0.0051	-0.0617	-1.0000	0.0340	-0.0347	-0.04527	
Misc	0.0022	0.0047	-0.0006	0.0138	-1.0000	0.0565	0.06091	
Contrib	0.0073	0.0075	0.0163	-0.0107	0.0530	-1.0000	0.12472	
Ins	0.0034	0.0026	0.0153	-0.0029	0.0097	0.0309	-1.00000	

As tests of stability of the Ψ_j vectors, regressions of one vector in a pair on the other have been undertaken on the sample of 40 survey-pairs analyzed earlier. In this case, the equations estimated have the form:

(3)
$$\Psi_{j} = \alpha + \beta \Psi_{k} + \varepsilon , \quad j \neq k.$$

Summaries of the results, following the format of Tables 7 and 8, are given in Tables 14 and 15. As with the equations for the ξ_i vectors based on expression (2), the things to look for in these tables are intercepts and slopes that are close to 0 and 1, respectively, and R^2 s that are close to 1. In Table 14, we see that mean intercepts deviate from 0 by less than a single standard deviation in every case and mean slopes from 1 for all but food. The results for food, housing, and personal insurance are especially striking. Mean R^2 s are greater than 0.97 for all three, with 38 of the 40 greater than 0.90 for housing and all 40 for food and insurance. Indeed, no R^2 is less than 0.985 for food or less than 0.96 for insurance. Other categories with a high frequency of R^2 s greater than 0.90 are transportation, health, entertainment, and tobacco. At the other extreme are alcoholic beverages, education, miscellaneous expenditures, for which mean R^2 s are less than 0.80 and only a handful of the individual R^2 s are greater than 0.90.

Table 13

Means of Budget Share T-Ratios

14 Categories of U.S. Consumption Expenditure

1996 Q1 - 2005 Q4

	<u>Categories</u>									
Coefficients	Food	<u>Alcbev</u>	<u>Housing</u>	<u>Apparel</u>	Trans	<u>Health</u>	<u>Entertn</u>			
Food		5.20	17.44	25.80	8.55	11.62	6.22			
Alcbev	4.91		2.80	2.05	1.55	-3.38	5.49			
Housing	32.24	2.28		7.09	6.75	9.52	8.80			
Apparel	10.01	2.64	7.23		4.07	-2.75	8.79			
Trans	8.05	1.65	5.14	5.59		2.70	4.97			
Health	11.10	-3.48	0.31	6.16	2.75		1.87			
Entertn	6.01	6.05	8.71	9.16	5.13	1.72				
Perscare	11.17	1.01	9.81	8.55	2.03	7.60	1.94			
Reading	2.66	3.15	4.25	4.66	0.84	9.63	9.07			
Educ	0.88	1.74	3.09	2.16	1.58	-1.17	2.01			
Tobacco	12.79	10.27	0.88	2.17	2.06	-0.04	1.60			
Misc	0.77	1.33	2.06	2.12	1.98	2.75	1.84			
Contrib	0.66	-0.71	1.61	1.71	1.38	4.99	2.77			
Ins	8.80	5.31	10.03	15.29	10.11	-3.67	7.64			

Table 13 cont'd

				Categories			
<u>Coefficients</u>	<u>Perscare</u>	Reading	Educ	<u>Tobacco</u>	Misc	<u>Contrib</u>	<u>Ins</u>
Food	10.99	2.57	1.10	12.50	0.77	0.65	8.80
Alcbev	0.86	3.01	1.79	10.27	1.29	-0.72	5.92
Housing	7.83	5.42	1.52	3.59	3.01	1.83	19.22
Apparel	10.58	3.48	3.63	-0.56	1.09	1.46	5.94
Trans	1.79	0.80	1.61	2.11	1.97	1.36	10.50
Health	7.82	9.23	-1.17	0.01	2.63	5.07	-3.50
Entertn	1.94	8.87	2.06	1.49	1.65	2.52	8.14
Perscare		4.70	0.09	-3.91	0.69	2.43	2.19
Reading	4.82		1.04	-1.00	1.82	4.30	2.05
Educ	0.18	1.15		-2.11	-0.01	1.07	2.54
Tobacco	-3.97	-1.06	-2.05		1.46	-1.39	-0.80
Misc	0.71	1.85	0.05	1.59		3.62	1.82

Contrib	2.45	4.26	1.00	-1.31	3.49		4.44
Ins	2.18	2.15	2.21	-0.82	1.77	4.34	

Table 14

Summary Statistics
Estimation of Expression (3)
40 Vector-Pairs
Budget Shares

		Ite	rcept		Slope			
			Mean	t-ratio			Mean	
<u>Category</u>	<u>Mean</u>	Std Dev	<u>t-ratio</u>	> 2	<u>Mean</u>	Std Dev	<u>t-ratio</u>	
				_				
Food	0.0055	0.007	0.97	5	0.92	0.07	51.96	
Alcoholic Beverages	0.0132	0.023	0.42	0	0.94	0.23	6.86	
Housing	0.0022	0.025	0.80	1	1.00	0.07	31.49	
Apparel	0.0060	0.017	0.39	1	0.94	0.26	11.63	
Transportation	-0.0009	0.003	-0.29	1	0.93	0.11	22.64	
Health	-0.0040	0.005	-0.46	0	0.99	0.16	17.45	
Entertainment	0.0036	0.008	0.28	1	0.98	0.15	14.64	
Personal Care	-0.0057	0.046	-0.02	0	0.96	0.20	10.46	
Reading	0.0299	0.056	0.28	1	0.88	0.21	8.75	
Education	0.0031	0.039	0.57	3	0.92	0.37	6.82	
Tobacco	0.0028	0.014	0.17	0	1.11	0.17	12.11	
Miscellaneous	0.0016	0.011	0.13	3	0.90	0.33	6.57	
Contributions	0.0070	0.008	0.55	0	0.76	0.26	5.59	
Insurance	-0.0016	0.004	-0.25	1	0.93	0.10	34.63	

Table 15 $R^2 s \ \ From \ Estimation \ of \ Expression \ (3)$ $40 \ \ Vector\ - Pairs$ $Budget \ Shares$

<u>Category</u>	<u>Mean</u>	<u>Std Dev</u>	<u>Min</u>	<u>Max</u>	$\underline{R^2} \ \geq \ 0.90$
Food	0.994	0.003	0.987	0.999	40
Alcoholic Beverages	0.761	0.123	0.477	0.947	5
Housing	0.975	0.029	0.889	0.998	38
Apparel	0.883	0.091	0.577	0.990	21
Transportation	0.941	0.089	0.589	0.997	33
Health	0.926	0.140	0.295	0.993	38
Entertainment	0.932	0.043	0.828	0.991	32
Personal Care	0.860	0.089	0.577	0.993	24
Reading	0.801	0.150	0.406	0.985	24
Education	0.743	0.143	0.403	0.982	4
Tobacco	0.914	0.049	0.757	0.977	31
Miscellaneous	0.703	0.210	0.042	0.971	5
Contributions	0.630	0.235	0.050	0.974	3
Insurance	0.987	0.008	0.960	0.997	40

IV. INTERPRETATION

At this point, we can say with some conviction that expenditures for food, housing, and personal insurance stand apart from other expenditures in households' budgets. There indeed appears to be something special about them. In the first part of this section, some suggestions are made as to what this might be. These will be pursued in two different frameworks. The first follows a line advanced in the *Third Edition* of *Consumer Demand in The United States* [Taylor and Houthakker (2010), hereafter *CDUS*), based upon work of the psychologist Abraham Maslow, that consumption behavior is determined according to a hierarchical ordering of preferences.¹⁴ The second framework is the one mentioned earlier of an interactive force field charged by income.

Over the course of a long and distinguished career, the psychologist Abraham Maslow put forth a compelling and influential argument that human behavior, at the most fundamental level, is motivated by five basic needs, consisting of physiological, security, love, self-esteem, and selfactualization. Physiological needs are the needs that must be fulfilled in order for the body to survive, which for our purposes will be taken to include food and drink, basic shelter, and sex. Once physiological needs are fulfilled, a new set of needs emerges -- security -- which can be described in terms of dependency, protection, freedom from fear, anxiety, and chaos, etc. Once physiological and security needs are satisfied, the needs of human beings as social animals come to the fore, beginning with needs for affection and community. Upon gratification of these needs, there arise a need or desire in people for a stable, firmly based, usually high evaluation of themselves, for selfrespect and the esteem of others. Finally, once physiological, security, belonging (love), and selfesteem needs are satisfied, the fifth and final basic motivation in the Maslow hierarchy emerges -self actualization -- which refers to people's desire for self-fulfillment, to become in fact what they are potentially. Figure 15 (which is taken from Chapter 2 of CDUS) provides a cross-classification schematic relating the five Maslovian needs to the individual consumption categories. For ease in presentation, the 14 categories are aggregated to six. Thick lines in the figure depict greater importance in satisfying a need than thin lines.

Figure 15

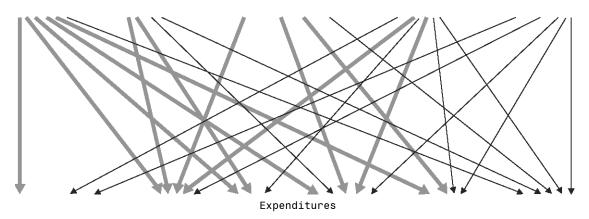
Cross-Classification of Maslov Hierarchy of Needs
And Consumption Categories

Maslovian Needs
Community & Affection Self-Esteem Self-Actualization

Security

Physiological

¹⁴ See Chapter 2 of *CDUS*.



As presented, these five Maslovian needs, form a rigid hierarchy, in which higher needs do not come into play until lower ones are satisfied. A model of consumption based upon these needs would accordingly be one in which tastes and preferences are assumed to be hierarchical, a world in which there is no indifference and wants are endless. However, as noted in *CDUS*, as a framework for organizing real-world consumer behavior, such an interpretation of Maslow's hierarchy is obviously too strict. Short of extreme physiological or security deprivation, an individual's consumption behavior at any point in time is probably best seen as being determined by whatever desire, at that moment, is most in need of gratification. Sometimes, this might be expenditures related to a physiological need, the installment of a security device to thwart potential burglary, or the purchase of artist supplies in pursuit of a long-term desire to paint. Indeed, for the very wealthy, it might be the purchase of a plot in a prestigious locale for the purpose of building a house that is larger than any of the individual's friends or acquaintances.

However, for the question at hand, if survival (physiological needs) and security are seen as the most basic (and most stable) of households' needs, the specialness of expenditures for food, housing, and personal insurance follows pretty much as a matter of course. Expenditures for food obviously serve physiological needs, insurance provides security, while housing provides for both. At the other end, we can associate the expenditure categories with least stability -- alcoholic beverages, education, miscellaneous, and contributions -- with the higher-order needs of community, self-esteem, and self-actualization. These are almost certainly idiosyncratic across individuals and households, and can account for their instability. For the other categories showing mean R²s in Table 15 greater than 0.90, expenditures for health can be associated with physiological and security needs and entertainment with community. Transportation serves a variety of ends, including survival (through a need to earn income) and security, which can account for its stability. Tobacco's apparent stability probably reflects a psychological need associated with addiction. Finally, for the remaining "in-between" categories -- apparel, personal care, and reading -- most of these expenditures can be attributed to the higher-order needs of community, self-esteem, and self-actualization.

The first framework (the one just described) views the Ψ vectors, in effect, as parameters associated with sets of hierarchical tastes and preferences. Rationalization in the second framework takes the view, in contrast, that the Ψ vectors represent points of "force" in a 14-dimensional "field" whose potential is fueled by income (or, as in present circumstances, total expenditure). Tastes and

preferences in this framework thus are characterized in terms of these points. The Ψ vectors associated with food, housing, and insurance, for this interpretation, accordingly represent points of strongest and most dominant force on expenditure allocation, while the vectors associated with alcoholic beverages, education, miscellaneous expenditures, and contributions represent points of least force. Vectors associated with transportation, health, entertainment, and tobacco exert less force than those associated with food, housing, and insurance, but more than the vectors associated with apparel, personal care, and reading.

As a final exercise in this section, we take a brief look at possible relationships between total expenditure and the Ψ vectors for the three "special" categories of food, housing, and personal insurance. The Ψ vectors for these categories have been re-estimated, for households with total expenditure above and below the median, respectively, for 1996q1, 1999q4, and 2005q4. The results, given in Tables 16 - 18, are quite clear in suggesting possibly significant Ψ -vector/total-expenditure relationships, especially for food. ¹⁵

V. INCOME AND SUBSTITUTION EFFECTS

As noted earlier, expenditure data collected in budget surveys reflect the conjunctive effects of income, prices, and tastes and preferences. While income data are collected in the CES surveys, price (or equivalently quantity) data are not, so that price effects can at best, using conventional procedures, only be adduced indirectly. However, since expenditures reflect the full ex post effects, not only of each category's own price, but of the effects of the prices of all other categories, it may be possible to eliminate the effects on expenditure of income effects resulting from price changes and thereby obtain estimates of pure "marginal propensities-to-consume". The basic idea is as follows. In the regressions for budget categories, we can interpret the intercepts (which are excluded from analysis of the budget coefficients) as representing the effects of income on categories after all effects of price and income on the other categories, as well as any interactive effects, are taken into account. The intercept for a category can accordingly be seen as representing a "pure" income effect that is net of any income effects arising from other categories. Category-specific "marginal propensities to consume" (with reference to total expenditure) can then be constructed by dividing

Food is especially of interest because of Engel's Law (which states that the income elasticity of food is positive but less than one). With Engel's Law, we might expect that, because the budget share for food varies inversely with total expenditure, the coefficients on food would be smaller for the upper 50th percentile than for the lower 50th percentile. In general, however, this is not the case, which suggests that the "food effect" on other expenditures is absolute rather than relative.

¹⁶ In *CDUS*, price data collected quarterly in surveys by the American Chambers of Commerce Research Association (ACCRA) were integrated with the CES surveys for 1996Q1 through 1999Q4, thereby allowing for the estimation of both price and income elasticities. This was done for an exhaustive breakdown of six consumption categories. It is not an easy task. See Chapter 6 of *CDUS* for details.

the difference in intercepts from different surveys by the difference in mean total expenditure. The results, based on mean differences for the 40 surveys between 1996Q1 and 2005Q4, are given in Table 19.¹⁷

Ordinarily, we think of negative marginal propensities to consume as reflecting inferior goods, but, it must be kept in mind that these are not ordinary MPCs, but rather MPCs after all of the effects arising from income on other expenditure categories are netted out. This being the case, the only entries in the table that might seem questionable are for transportation, for which the value is -0.11 and tobacco, which shows a positive value (though extremely small). The "big 3" in terms of stability of Ψ vectors (food, housing, and personal insurance), interestingly, are also the ones with the largest MPCs.

	1996Q1		1999	9Q4	2005Q4		
<u>Category</u>	Upper 50%	Lower 50%	<u>Upper 50%</u>	Lower 50%	<u>Upper 50%</u>	Lower 50%	
Alcoholic Beverages	0.0305	0.0086	0.0244	0.0115	0.0205	0.0039	
	(4.95)	(1.50)	(6.34)	(3.55)	(5.71)	(1.15)	
Housing	0.7535	0.7525	0.8472	0.6890	1.0196	0.8775	
	(13.61)	(16.28)	(19.51)	(22.07)	(25.09)	(29.57)	
Apparel	0.0819	0.0481	0.0970	0.0445	0.0706	0.0411	
	(4.31)	(3.38)	(9.34)	(6.55)	(9.12)	(6.73)	
Transportation	0.2657	0.1730	0.3895	0.1267	0.4509	0.2068	
	(3.45)	(6.77)	(7.21)	(8.44)	(10.19)	(14.72)	
Health	0.1192	0.1064	0.1441	0.1618	0.1739	0.0513	
	(4.74)	(4.82)	(8.36)	(10.98)	(9.90)	(1.98)	
Entertainment	0.0792	0.0511	0.1017	0.0344	0.0803	0.0445	
	(3.70)	(4.02)	(7.56)	(4.83)	(6.12)	(7.54)	
Personal Care	0.0183	0.0204	0.0183	0.0199	0.0092	0.0127	
	(6.54)	(5.12)	(9.02)	(8.55)	(5.45)	(7.28)	
Reading	0.0051	0.0022	0.0018	0.0004	0.0044	0.0012	
	(2.07)	(0.77)	(1.09)	(0.24)	(3.85)	(1.14)	
Education	0.0482	0.0013	0.0346	0.0050	0.0718	0.0053	
	(2.12)	(0.09)	(2.77)	(0.60)	(4.08)	(0.79)	

¹⁷ The results in Table 19 refer to calculations between adjacent surveys.

Tobacco	0.0317	0.0379	0.0396	0.0397	0.0257	0.0337
	(6.19)	(5.47)	(9.61)	(7.57)	(6.43)	(7.05)
Miscellaneous	0.0306	-0.0071	0.0161	0.0055	0.0130	0.0011
	(1.37)	(-0.66)	(1.50)	(1.13)	(1.28)	(0.27)
Contributions	-0.0144	0.0043	0.0162	0.0081	0.0402	0.0372
	(-0.97)	(0.71)	(1.54)	(2.06)	(2.75)	(4.84)
Insurance	0.1363	0.0815	0.3060	0.1175	0.3273	0.1120
	(3.60)	(3.42)	(11.51)	(8.64)	(14.65)	(8.88)
Mean total expenditure Median total expenditure No. of observations	\$13,551 \$71	\$4482 190 173	·	\$4329 212 825	·	\$5327 714 640

Table 17

Budget Share Coefficients on Housing Upper and Lower Median Percentiles CES Surveys 1996Q1, 199Q4, 2005Q4 (t-ratios in parentheses)

	1996Q1		199	9Q4	2005Q4		
<u>Category</u>	<u>Upper 50%</u>	Lower 50%	Upper 50%	Lower 50%	Upper 50%	Lower 50%	
Food	0.1844	0.2473	0.1407	0.2144	0.1448	0.2214	
	(13.68)	(16.28)	(19.51)	(22.07)	(25.07)	(29.57)	
Alcoholic Beverages	0.0059	-0.0003	0.0032	0.0034	0.0040	0.0009	
	(1.93)	(-0.10)	(2.01)	(1.90)	(2.96)	(0.51)	
Apparel	0.0285	0.0365	0.0176	0.0275	0.0170	0.0088	
	(3.02)	(4.49)	(4.10)	(7.28)	(5.78)	(2.86)	
Transportation	0.1363	0.0582	0.1008	0.0646	0.0873	0.0615	
	(3.57)	(3.92)	(4.55)	(7.70)	(5.18)	(8.55)	
Health	0.0422	0.0825	0.0343	0.0699	0.0361	0.0930	
	(3.37)	(6.57)	(4.84)	(8.42)	(5.40)	(7.19)	
Entertainment	0.0452	0.0247	0.0292	0.0253	0.0267	0.0277	
	(4.27)	(3.38)	(5.30)	(6.40)	(5.40)	(9.40)	
Personal Care	0.0058	0.0057	0.0077	0.0053	0.0048	0.0068	
	(4.17)	(2.47)	(9.38)	(4.01)	(7.57)	(7.73)	
Reading	0.0038	0.0051	0.0040	0.0033	0.0008	0.0015	
	(3.13)	(3.15)	(5.87)	(3.78)	(1.93)	(2.94)	
Education	0.0112	0.0010	0.0172	-0.0009	0.0067	0.0026	
	(1.00)	(0.12)	(3.38)	(-0.21)	(1.01)	(0.77)	
Tobacco	0.0017	0.0063	0.0010	0.0068	0.0036	0.0080	

	(0.65)	(1.57)	(0.57)	(2.31)	(2.38)	(3.30)		
Miscellaneous	0.0047	0.0141	0.0178	0.0024	0.0083	0.0029		
	(0.43)	(2.29	(4.09)	(0.89)	(2.16)	(1.45)		
Contributions	0.0147	-0.0009	-0.0018	-0.0005	0.0164	0.0001		
	(2.01)	(-0.27)	(-0.42)	(-0.23)	(2.97)	(0.03)		
Insurance	0.1939	0.0833	0.1496	0.0738	0.1339	0.0709		
	(10.80)	(6.17)	(13.95)	(9.76)	(15.99)	(11.26)		
Mean total expenditure	\$13,551	\$4482	\$14,534	\$4329	\$17,753	\$5327		
Median total expenditure	\$71	90	\$	7212	\$8	\$8714		
No. of observations	11	73		2825	3	3640		

Table 18

Budget Share Coefficients on Personal Insurance
Upper and Lower Median Percentiles
CES Surveys
1996Q1, 199Q4, 2005Q4
(t-ratios in parentheses)

	1996Q1		1999	9Q4	2005Q4		
<u>Category</u>	Upper 50%	Lower 50%	<u>Upper 50%</u>	Lower 50%	<u>Upper 50%</u>	Lower 50%	
Food	0.0810	0.1225	0.1470	0.2202	0.1707	0.1901	
Food	(3.60)	(3.42)	(11.51)	(8.64)	(14.65)	(8.88)	
	(3.00)	(3.42)	(11.51)	(8.04)	(14.03)	(0.00)	
Alcoholic Beverages	0.0029	0.0258	0.0067	0.0143	0.0073	0.0230	
	(0.60)	(3.66)	(2.49)	(3.21)	(2.79)	(5.23)	
Housing	0.4712	0.3814	0.4327	0.4444	0.4916	0.4769	
· ·	(10.80)	(6.17)	(13.95)	(9.76)	(15.99)	(11.26)	
Apparel	0.0309	0.0324	0.0164	0.0243	0.0080	0.0250	
	(2.09)	(1.85)	(2.25)	(2.60)	(1.42)	(3.12)	
Transportation	0.0724	0.2675	0.0919	0.2405	0.1344	0.2501	
Transportation	(1.21)	(8.64)	(2.44)	(11.84)	(4.16)	(13.61)	
Health	-0.0092	0.1113	-0.0131	-0.1469	0.0083	0.1164	
пеатт							
	(-0.47)	(-4.10)	(-1.08)	(-7.19)	(0.65)	(3.45)	
Entertainment	0.0457	0.0366	0.0377	0.0195	0.0484	0.0161	
	(2.76)	(2.34)	(4.01)	(2.00)	(5.09)	(2.08)	
Personal Care	0.0046	0.0038	0.0030	0.0070	0.0060	-0.0076	
	(2.11)	(0.77)	(2.09)	(2.16)	(4.93)	(-3.31)	
Reading	0.0111	-0.0012	0.0054	-0.0010	0.0006	-0.0026	
	(5.92)	(-0.34)	(4.64)	(-0.46)	(0.71)	(-1.95)	
	(-)	,/	(/	,,	()	, /	

Education	0.0037	-0.0145	0.0081	-0.0003	0.0387	-0.0036		
	(0.21)	(-0.82)	(0.93)	(-0.02)	(3.04)	(-0.41)		
Tobacco	-0.0005	0.0076	0.0006	0.0164	-0.0021	0.0019		
	(-0.13)	(0.89)	(0.20)	(2.26)	(-0.73)	(0.30)		
Miscellaneous	-0.0081	0.0099	-0.0037	0.0077	0.0104	0.0128		
	(-0.47)	(0.75)	(-0.50)	(1.16)	(1.41)	(2.49)		
Contributions	0.0100	0.0092	0.0148	0.0047	0.0525	0.0230		
	(0.88)	(1.25)	(2.03)	(0.87)	(4.98)	(2.29)		
Mean total expenditure	\$13,551	\$4482	\$14,534	\$4329	\$17,753	\$5327		
Median total expenditure	\$7	190	\$7	212	, \$8	\$8714		
No. of observations	1173		2	2825	3640			

Table 19

Mean Pure Marginal Propensities to Consume
14 Categories of Expenditure
CES Budget Surveys
1996 Q1 - 2005 Q4

<u>Category</u>	<u>MPC</u>
Food	0.0586
Alcoholic Beverages	-0.0066
Housing	0.1149
Apparel	-0.0136
Transportation	-0.1113
Health	0.0509
Entertainment	-0.0105
Personal Care	-0.0090
Reading	-0.0058
Education	-0.0486
Tobacco	0.0033
Miscellaneous	-0.0020
Contributions	-0.0153
Insurance	0.0721

We now turn to matters of substitution and complementarity. In the conventional analysis of consumer behavior, these concepts are approached in terms of signs of cross-price elasticities. A good is considered a substitute for another good if the sign of their cross-price elasticity is negative and a complement if the sign is positive. While we do not possess price elasticities in the present context, we do have their representations in expenditure and budget share coefficients. However, since these coefficients reflect income as well as price effects, substitution and complementarity must now be interpreted as representing the combined effects of income and prices. With this proviso in

mind, let us turn attention to Table 20, which tabulates, for each category of expenditure, complements and substitutes as determined by signs (positive for complements, negative for substitutes) of the coefficients in Table 15.¹⁸ Complementarity (on this definition) is seen to be total for food, housing, transportation, and entertainment, and nearly so for personal care, reading, and miscellaneous expenditures. The only categories with more than two other categories depicted as substitutes are tobacco (with six) and health (with five). The results for tobacco would appear to be Table 20

Substitutes and Complements
14 Categories of Expenditure
CES Budget Surveys
1996 Q1 - 2005 Q4

0-4----

	<u>Categories</u>										
	<u>Food</u>	<u>Alcbev</u>	<u>Housing</u>	<u>Apparel</u>	<u>Trans</u>	<u>Health</u>	<u>Entertn</u>				
Complements	All	food housing apparel trans entertn perscare reading educ tobacco misc	All	food alcbev housing trans entertn perscare reading educ misc contrib ins	All	food housing trans entertn perscare reading misc contrib	A11				
Substitutes		health contrib		health tobacco		alcbev apparel educ tobacco ins					
			C	ategories							
	<u>Perscare</u>	Reading	Educ	<u>Tobacco</u>	<u>Misc</u>	<u>Contrib</u>	<u>Ins</u>				
Complements	All but tobacco	All but tobacco	food alcbev housing apparel trans entertn perscare reading	food alcbev housing trans entertn misc	All but educ	food housing apparel trans health entertn perscare reading	food alcbev housing apparel trans entertn perscare reading				

Complements can arise on this definition simply because both are normal goods and positive income effects more than offset any negative cross-price effects. Similarly, goods can appear as substitutes because negative cross-price effects offset positive income effects or, alternatively, because one of the goods is inferior. Tobacco, as discussed in the next footnote, would seem a good example.

60

			contrib ins			educ misc ins	educ misc contrib ins
Substitutes	tobacco	tobacco	health tobacco misc	apparel health perscare reading educ contrib	educ	alcbev tobacco	health tobacco

a reflection of complex income effects arising from the negative consequences of smoking.¹⁹ The results for health, on the other hand, are almost certainly influenced by the fact that the Bureau of Labor Statistics treats health services reimbursed by insurance as negative expenditures. Effects of this will be discussed in the next section.

VI. SOME TECHNICAL CONSIDERATIONS RELATING TO DATA AND ESTIMATION

A side benefit of the analyses in this exercise is that there is sufficient stability and structure in the ξ vectors to detect anomalies in the BLS surveys that might otherwise be hidden from view. When coefficients in a ξ_i vector for quarter stand out from their counterparts for other quarters, something is usually amiss. A case in point is, as just noted, the fact that BLS treats insurance-reimbursed services as negative expenditures. Unfortunately, I was not aware of this until I was preparing the graphs for the coefficients for health and insurance in Figures 6 and 14, which showed the estimated coefficients on insurance in Figure 6 and on health in Figure 14 for 2005Q2 and 2005Q4 as wildly out of line with estimates for the other quarters. This led (unfortunately for the first time) to a detailed examination of the data for health expenditures and to find negative entries for some households as result of insurance-reimbursed services being recorded as negative expenditures. The equations for these quarters, as well as for the first quarter of 2001 and the first three quarters of 2002, were re-estimated with the households with insurance-reimbursed health services removed from the sample. The re-estimated coefficients for these quarters then fell in line with their sisters.²⁰

Another category in which care needs to be taken in interpreting the results is transportation.

¹⁹ I say "complex", because, while social and health circumstances clearly view tobacco as an inferior good, total-expenditure elasticities estimated from CES data nevertheless turn out to be positive. (Cf., Chapter 11 of *CDUS*, for the 16 CES surveys for 1996Q1 through 1999Q4).

There remains, of course, a question of how insurance-reimbursed expenditures should be treated in consumption analysis. Exclusion of the households affected, as has been done here, is a short-term fix occasioned by the problem not being discovered until late in the exercise.

In this case, the problem is not with BLS methodology, per se, but rather with the fact that vehicle purchases are involved. As automobiles are purchased infrequently, most households will record zero values for their purchase, which means that, for most households in a survey, transportation expenditures will consist of a flow of expenditures for gas, oil, insurance, public transportation, etc. However, for households in a survey that do record non-zero expenditures for vehicle purchase, expenditures for transportation will be elevated (perhaps severely so) in relation to what they would otherwise be. The distortion in expenditures that this creates almost certainly accounts for much of the variability in the estimated coefficients that is observed in the graphs for transportation in Figure 5.²¹

Finally, let us turn attention to estimation questions, particularly with regard to the regression equations for budget shares. Since the dependent and independent variables in these equations form an exhaustive set of budget shares, they obviously sum to 1. An equation is therefore redundant, which means that the estimated coefficients for any one of the equations can be deduced from estimates from the other 13. However, the easiest way of obtaining the t-ratios and R² for the redundant equation is to estimate it as well. Estimation would only be a problem if the equations were to be estimated as a system of seemingly unrelated regressions.²² While there are certainly reasons to think that the error terms across equations are correlated, joint estimation has not been pursued.

Still another consideration relates to the fact that the roles of variables as dependent and independent variables are exchanged. In the equation for food, for example, housing is an independent variable, while in the equation for housing, food is an independent variable. While this creates evident simultaneity, it is a problem in this situation for which there is no solution, for there are no exogenous variables. Biases undoubtedly exist in the estimates, but are hopefully constant across surveys. Beyond this is a question of how the coefficient on housing in the equation for food is related to the coefficient on food in the equation for housing, and similarly for the other related pairs. One might think them to be reciprocals, but a glance at Tables 1 and 2 shows that this is decidedly not the case. What is true is that the estimated coefficients for any related pair share the same t-ratio ²³

A final contingency to be mentioned is the fact that the CES surveys are semi-panel in nature,

This problem is most acute for vehicle purchases, but is present to some extent in any category in which durable goods are involved. Interestingly, its recognition here leads to an explanation of the anomalous result obtained in Chapter 6 of *CDUS*, in which it was found that the two largest principal components for the 14 categories of expenditure, in essence, differ only in the sign on transportation expenditures. The principal components, in effect, segregate households that purchased a vehicle from those that did not.

The covariance matrix of the error terms of the joint system of 14 equations would be singular, and only 13 equations could be estimated.

²³ Cf., Tables 1 and 2.

in that once a household (or consumer unit in BLS terminology) is selected for inclusion, it remains in the survey for five consecutive calendar quarters. Entry and exit of households is staggered so that each quarter's sample consists of a mixture of "new" and "old" households. In view of this, we might reasonably expect to find greater stability in the ξ and Ψ vectors in adjacent surveys than in surveys separated by more than five quarters.

VII. A FORECASTING EXERCISE

As a final exercise, the mean ξ_i vectors from Table12 will be used to make predictions of mean expenditures and budget shares in the four CES surveys for 2006, the purpose being to see whether the information in these vectors offer predictive power beyond that is provided in the standard format of regressions of expenditures in separate categories on income (or total expenditure) and a variety of socio-demographic characteristics. The answer turns out to be no. Predictions have been derived as follows:

- (1). The observed mean total expenditure for a quarter is assumed to be given, which means that this total is allocated amongst the 14 categories of expenditure.
- (2). Total expenditure is multiplied by the vector of mean budget shares (given in Table 21) from the 40 surveys for 1996 through 2005 to provide a first estimate of category expenditures according to:

(4)
$$z_i = (a_i + k_i)Z$$
, $i = 1, ..., 14$,

where z_i denotes expenditures in the i^{th} category, Z denotes total expenditure, k_i is the mean budget share for the category over the 40 surveys for 1996 through 2005, and a_i represent mean intercept coefficients defined as the ratio of the mean intercept in the budget category regressions to mean total expenditure (both calculated over the 40 surveys). These intercepts are given in the first column of Table 21.

(3). The intercepts are adjusted in subsequent iterations until the sum of the predictions from expression (2) equals total expenditure. The results are tabulated in Table 22.

In Table 22, we see that predictions are excellent for food, alcoholic beverages, and personal insurance, generally very good for housing, transportation, and tobacco, and not at all good for reading, education, miscellaneous, and contributions. In general, this is in keeping with the stability that has been found in the ξ_i vectors for food, housing, and personal insurance, and instability for reading, miscellaneous, and contributions. As a measure of "fit", regressions have been estimated in which the actual budget shares in Table 22 for each quarter are regressed on the predicted values, with results as given in Table 23. Similar regressions have also been estimated for predictions based solely on the mean budget shares for the 40 "historical" surveys. In every case, R^2 s are high,

intercepts are essentially 0, and slope coefficients are either within or very close to being within a single standard deviation of 1. Interestingly, however, predictions derived from just the mean budget shares turn out to be superior to those derived from the mean intra-budget-share coefficients.

As a final forecasting exercise, predictions of expenditures for individual households in the four 2006 surveys are constructed by multiplying each household's total expenditure by the predicted budget shares in Table 22. Measures of fit are then obtained by regressing actual expenditures on the estimates, as in Table 23. The results are given in Table 24. Predictions are seen to be best for food, housing, transportation, and personal insurance, and worst for tobacco and miscellaneous expenditures, none of which, at this point, should be surprising. On the other hand, what might appear surprising is that, while estimated coefficients differ between the predicted and mean budget shares, *t-ratios and R*²s are seen to be identical. However, before examining the reason for this

Table 21

Mean Intercept Coefficients and Budget Shares
14 Categories of Expenditures

CES Budget Surveys
1996 Q1 - 2005 Q4

	Intercept		Budget Share				
<u>Category</u>	<u>Coefficient</u>	Mean	Std Dev	<u>Minimum</u>	<u>Maximum</u>		
Food	0.0732	0.1376	0.0058	0.1248	0.1476		
Alcbev	0.00044	0.0085	0.0005	0.0075	0.0094		
Housing	0.1154	0.3183	0.0053	0.3053	0.3305		
Apparel	-0.0072	0.0365	0.0079	0.0261	0.0549		
Trans	0.0573	0.1997	0.0105	0.1789	0.2198		
Health	0.0031	0.0538	0.0037	0.0484	0.0608		
Entertn	-0.0054	0.0528	0.0040	0.0457	0.0615		
Perscare	0.0022	0.0076	0.0012	0.0057	0.0091		
Reading	0.00068	0.0042	0.0009	0.0027	0.0059		
Educ	-0.0081	0.0168	0.0037	0.0115	0.0264		
Tobacco	0.0071	0.0077	0.0006	0.0066	0.0092		
Misc	-0.0015	0.0144	0.0021	0.0111	0.0190		
Contrib	-0.0015	0.0215	0.0135	0.0079	0.0471		
Insurance	0.0102	0.1205	0.0031	0.1133	0.1285		

Table 22

Predicted Expenditures and Budget Shares
14 Categories of Expenditure
CES Surveys
2006 Q1 - 2006 Q4

			Expe	enditures					
	2006	2006q1		2006q2		2006q3		2006a4	
Category	<u>Actual</u>	Pred.	<u>Actual</u>	Pred.	<u>Actual</u>	Pred.	<u>Actual</u>	Pred.	

Food	\$1504	\$1561	\$1610	\$1564	\$1447	\$1494	\$1636	\$1605
Alcbev	96	100	101	107	103	104	103	103
Housing	3855	3979	4190	4137	3552	3574	3798	3748
Apparel	440	300	320	342	480	448	434	437
Trans	2032	2240	2398	2279	2467	2551	2663	2638
Health	676	697	696	716	593	603	665	651
Entertn	667	595	608	602	786	778	767	770
Perscare	77	75	76	78	113	116	121	120
Reading	37	31	30	33	61	61	62	62
Education	219	180	212	281	263	250	234	238
Tobacco	78	84	87	88	62	76	90	87
Misc	155	192	153	153	136	133	130	130
Contrib	551	483	483	481	312	290	277	280
Ins	1435	1453	1475	1464	1457	1439	1459	1454

Total Expend: \$11,823 \$11,918 \$12,439 \$12,324

Table 22 cont'd

	Budget Shares										
	Mean	2006	3q1	2006q2		2006	5q3	2006	6a4		
Category	1996-2005	<u>Actual</u>	Pred.	<u>Actual</u>	Pred.	<u>Actual</u>	Pred.	<u>Actual</u>	<u>Pred.</u>		
Food	0.1376	0.1272	0.1224	0.1310	0.1254	0.1294	0.1316	0.1269	0.1302		
Alcbev	0.0085	0.0081	0.0087	0.0084	0.0087	0.0081	0.0083	0.0087	0.0084		
Housing	0.3183	0.3261	0.3004	0.3338	0.2999	0.3368	0.3053	0.3357	0.3041		
Apparel	0.0365	0.0372	0.0406	0.0252	0.0376	0.0257	0.0349	0.0277	0.0355		
Trans	0.1997	0.1719	0.2087	0.1879	0.2141	0.1928	0.2140	0.1849	0.2140		
Health	0.0538	0.0572	0.0502	0.0585	0.0506	0.0560	0.0535	0.0581	0.0529		
Entertn	0.0528	0.0565	0.0665	0.0499	0.0653	0.0489	0.0617	0.0488	0.0625		
Perscare	0.0076	0.0065	0.0096	0.0063	0.0098	0.0061	0.0097	0.0063	0.0097		
Reading	0.0042	0.0032	0.0052	0.0026	0.0051	0.0024	0.0050	0.0026	0.0050		
Education	0.0168	0.0185	0.0222	0.0151	0.0210	0.0170	0.0188	0.0228	0.0193		
Tobacco	0.0077	0.0066	0.0052	0.0070	0.0064	0.0070	0.0072	0.0072	0.0070		
Misc	0.0144	0.0132	0.0115	0.0161	0.0111	0.0123	0.0104	0.0124	0.0106		
Contrib	0.0216	0.0466	0.0263	0.0363	0.0243	0.0388	0.0223	0.0390	0.0227		
Ins	0.1205	0.1213	0.1232	0.1219	0.1208	0.1186	0.1173	0.1188	0.1180		

Table 23

Regressions of Actual on Predicted Values Budget Shares CES 14 Categories 2006 Q1 - 2006 Q4 (From Table 22)

	Predicted Budget Shares			Mean Budget	Shares,	1996-2005
<u>Quarter</u>	<u>Coefficient</u>	Std.Dev	R2	Coefficient	Std.Dev	R2
2006 Q1	1.0047	0.0463	0.9751	0.9688	0.0334	0.9859
2006 Q2	1.0435	0.0433	0.9798	1.0164	0.0241	0.9933
2006 Q3	1.0396	0.0377	0.9845	1.0268	0.0248	0.9930
2006 Q4	1.0242	0.0429	0.9794	1.0093	0.0285	0.9905
2006 Q1 2006 Q2 2006 Q3	1.0047 1.0435 1.0396	0.0463 0.0433 0.0377	0.9751 0.9798 0.9845	0.9688 1.0164 1.0268	0.0334 0.0241 0.0248	0.9859 0.9933 0.9930

Table 24

Regressions of Actual on Predicted Values
Households
CES 14 Categories
2006 Q1 - 2006 Q4

	Predicted Budget Shares			Mean Budget Shares, 1996-2005		
<u>Category</u>	<u>Coefficient</u>	<u>t-ratio</u>	<u>R2</u>	<u>Coefficient</u>	<u>t-ratio</u>	R2
Food						
2006 Q1	0.4729	58.76	0.3073	0.4373	58.76	0.30
2006 Q2	0.4124	52.40	0.2816	0.3926	52.40	0.28
2006 Q3	0.4974	53.53	0.2908	0.4677	53.53	0.29
2006 Q4	0.4751	55.55	0.3035	0.4380	55.55	0.30
Alcoholic Beverages		00100	010000	011000	00100	0.00
2006 Q1	0.7637	26.25	0.0813	0.7251	26.25	0.08
2006 Q2	0.6934	23.13	0.0709	0.6793	23.13	0.07
2006 Q3	0.7722	26.36	0.0904	0.7355	26.36	0.09
2006 Q4	0.7806	26.02	0.0885	0.7981	26.02	0.08
Housing	0.7000	20.02	0.0000	017001	20.02	0.00
2006 Q1	0.7394	88.25	0.5001	0.7575	88.25	0.50
2006 Q2	0.7394	81.20	0.4848	0.7468	81.20	0.48
2006 Q2 2006 Q3	0.8617	91.93	0.4646	0.7408	91.93	0.54
2006 Q3 2006 Q4	0.8323	83.32	0.4890	0.9778	83.32	0.48
Apparel	0.6323	65.52	0.4690	0.6777	65.52	0.40
2006 Q1	1.2183	45 27	0.2000	1.2436	45 27	0.00
2006 Q1 2006 Q2	1.0826	45.37 46.74	0.2090 0.2377	0.7482	45.37 46.74	0.20
2006 Q2 2006 Q3						0.23
2006 Q3 2006 Q4	1.0319	40.84	0.1928	0.7274	40.84	
	0.9827	44.26	0.2182	0.7468	44.26	0.21
Transportation 2006 Q1	1 7067	74 40	0 4161	1 4050	74 40	0 41
	1.7367	74.48	0.4161	1.4950	74.48	0.41
2006 Q2	1.9055	83.74	0.5002	1.7934	83.74	0.50
2006 Q3	1.5469	72.19	0.4273	1.4935	72.19	0.42
2006 Q4	1.5819	72.82	0.4282	1.4650	72.82	0.42
Health	0 5000	07.00	0.0000	0.5057	07.00	0.00
2006 Q1	0.5326	27.63	0.0893	0.5657	27.63	0.08
2006 Q2	0.4557	23.60	0.0736	0.4950	23.60	0.07
2006 Q3	0.4764	24.19	0.0817	0.4954	24.19	0.08
2006 Q4	0.5344	26.79	0.0919	0.5765	26.79	0.09
ole 24 cont'd						
Entertainment						
2006 Q1	1.2088	46.43	0.2168	1.2928	46.43	0.21
2006 Q2	1.5558	34.50	0.1452	1.4700	34.50	0.14
2006 Q3	1.2368	41.33	0.1965	1.1456	41.33	0.19
2006 Q4	1.2877	39.29	0.1790	1.1910	39.29	0.17
Personal Care						
2006 Q1	0.6687	36.97	0.1494	0.5703	36.97	0.14
2006 Q2	0.6785	36.66	0.1609	0.5647	36.66	0.16
2006 Q3	0.7637	38.22	0.1728	0.6130	38.22	0.17
2006 Q4	0.7279	35.46	0.1508	0.6041	35.46	0.15
Reading						
2006 Q1	0.7130	27.67	0.0895	0.5303	27.67	0.08
2006 Q2	0.6491	24.91	0.0814	0.3993	24.91	0.08
2006 Q3	0.7143	27.37	0.0968	0.4069	27.37	0.09

2006 Q4	0.6237	24.73	0.0795	0.3880	24.73	0.0795
Education						
2006 Q1	1.8796	27.81	0.0904	2.0745	27.81	0.0904
2006 Q2	1.1650	24.87	0.0810	1.9526	24.87	0.0810
2006 Q3	2.2934	29.81	0.1129	2.3284	29.81	0.1129
2006 Q4	1.9144	27.09	0.0939	2.6059	27.09	0.0939
Tobacco						
2006 Q1	0.0516	1.66	0.0004	0.0445	1.66	0.0004
2006 Q2	0.0189	0.57	0.0000	0.0172	0.57	0.0000
2006 Q3	0.0650	1.98	0.0006	0.0594	1.98	0.0006
2006 Q4	0.0249	0.75	0.0001	0.0232	0.75	0.0001
Miscellaneous						
2006 Q1	1.3780	20.12	0.0495	1.2581	20.12	0.0495
2006 Q2	1.0256	18.07	0.0445	1.1466	18.07	0.0445
2006 Q3	1.1632	18.07	0.0446	0.9920	18.07	0.0446
2006 Q4	1.2719	17.71	0.0424	1.0965	17.71	0.0424
Contributions						
2006 Q1	1.8502	41.00	0.1776	3.9980	41.00	0.1776
2006 Q2	1.7503	37.73	0.1689	2.9460	37.73	0.1689
2006 Q3	1.7860	33.13	0.1358	3.2172	33.13	0.1358
2006 Q4	2.1993	34.77	0.1458	3.9823	34.77	0.1458
Insurance						
2006 Q1	0.8570	64.45	0.3480	0.8629	64.45	0.3480
2006 Q2	0.7791	56.95	0.3164	0.7799	56.95	0.3164
2006 Q3	0.8242	60.05	0.3405	0.8111	60.05	0.3405
2006 Q4	0.7420	58.71	0.3274	0.7316	58.71	0.3274

Table 25

Regressions on Total Expenditure

CES 14 Categories

2006 Q1

	Slope		
<u>Category</u>	<u>Coefficient</u>	<u>t-ratio</u>	R2
Food	0.0602	58.76	0.3073
Alcoholic Beverages	0.0062	26.25	0.0813
Housing	0.2411	88.25	0.5001
Apparel	0.0454	45.37	0.2091
Transportation	0.2985	74.48	0.4161
Health	0.0305	27.63	0.0893
Entertainment	0.0683	46.43	0.2168
Personal Care	0.0043	36.97	0.1494
Reading	0.0023	27.67	0.0895
Education	0.0348	27.81	0.0904
Tobacco	0.0003	1.66	0.0004
Miscellaneous	0.0181	20.12	0.0495
Contributions	0.0862	41.00	0.1776
Insurance	0.1040	64.45	0.3480

result, it will be useful to see how this procedure for predicting expenditure allocation compares with the more traditional Engel-Curve format, in which expenditures in a category are regressed on total expenditure. The results for 2006Q1 are presented in Table 25. As can be seen, t-ratios and R²s are identical with those in Table 24. With hindsight, the explanation is simple. *Total expenditure is the driver in both cases, directly in the regression equations in Table 24 and as homogeneous linear combinations in the equations using budget-share predictions in Table 23*.

VIII. CONCLUSION

It is time to bring this investigation into the internal structure of U. S. consumption expenditures to a close. The study has proceeded in a framework in which expenditure categories are viewed as vectors in a space of the same dimension as the number of categories. Coordinates of the vectors (whether defined in terms of expenditures or as budget shares) are viewed as representing the mutually interactive influences that expenditures across different categories have on one another. Concern has been with stability of coefficients in the vectors, based on data from the 40 CES surveys from 1996 through 2005. Coefficients associated with food, housing, and personal insurance are found to be the most stable, while those associated with alcoholic beverages, education, miscellaneous expenditures, and contributions are the least stable.²⁴

An important question, of course, is what interpretation is to be put on these instabilities. In the first place, are they inherent features of consumer tastes and preferences? Or do they simply represent stability in the overall distribution of consumption patterns across U. S. households? Almost certainly, there would seem to be elements of both. Standard neoclassical theory of consumer choice is, of course, premised on the former, while 10 years is obviously too short a period for major distributional shifts to become apparent. However, in my view, the most interesting result emerging from the study is the possible existence of "category" effects (which may vary with the level of income) that affect expenditures in other categories asymmetrically. Food, housing, and personal insurance appear to be candidates. As described in Section IV, such effects can be straightforwardly interpreted in theoretical framework founded on a Maslovian hierarchy of needs.

While it was hoped that the stabilities identified might be useful in complementing traditional Engel-Curve procedures for forecasting the allocation of consumption expenditures amongst different categories, this turns out (as we have just seen) not to be the case. For the information embedded in the forecasting procedures suggested (based upon historical budget shares and intra-budget relationships) contains identically the same information as that provided by total expenditure.

In closing, the message that I most wish to leave with readers is that there is indeed a great deal of stability in the internal structure of U. S. consumption behavior, especially as embodied in the important categories (in monetary terms) of food, housing, transportation, and personal insurance. Not only are expenditures in these categories stable in terms of their own budget shares, but also in relation to other categories. Delving into why these relationships exist, and why they persist, would seem an interesting new area of research. An evident place to begin would be to apply the methods of this study to much earlier surveys of consumer expenditure, specifically, to the BLS surveys of 1961 and 1971-72.

²⁴ If dynamical effects arising from the purchase of motor vehicles were to be properly taken into account, coefficients for transportation expenditures would probably be found to be stable as well.

Appendix 1

Data and Definitions

All data in the study are taken from the public use microdata CD-ROMs of Consumer Expenditure, 1996-2006, obtained from the Bureau of Labor Statistics, U.S. Department of Labor, specially, from the *finli* data files.²⁵ Data from 40 quarters of surveys (1996Q1 through 2005Q4 are analyzed in all. The numbers of households represented in the surveys are given in Table 1²⁶

Table 1

Households in The 40 CES Surveys Analyzed
1996 Q1 - 2005 Q4

<u>Quarter</u>	<u>Households</u>	<u>Quarter</u>	<u>Households</u>
1996Q1	2346	2001Q1	5562
1996Q2	3594	2001Q2	5841
1996Q3	3540	2001Q3	5836
1996Q4	3657	2001Q4	6147
1997Q1	3709	2002Q1	6169
1997Q2	3743	2002Q2	6300
1997Q3	3789	2002Q3	6356
1997Q4	3808	2002Q4	6500
1998Q1	3795	2003Q1	6498
1998Q2	3808	2003Q2	6690
1998Q3	3678	2003Q3	6640
1998Q4	3653	2003Q4	6592
1999Q1	4587	2004Q1	7632
1999Q2	5004	2004Q2	7522
1999Q3	4855	2004Q3	7386
1999Q4	4847	2004Q4	7372
2000Q1	5815	2005Q1	4874
2000Q2	5801	2005Q2	7379
2000Q3	5670	2005Q3	7199
2000Q4	5610	2005Q4	7242

The 14 exhaustive expenditure categories (with abbreviations) that are analyzed in the study are listed in Table 2.

²⁵ A detailed description of the surveys can be found in Chapter 17 of the BLS Handbook of Methods (1997, pp. 161-164). See, also, Chapter 4 of Taylor and Houthakker (2010).

Households with incomes less than \$5000 and 0 or negative total expenditure are excluded from the analysis. Households with negative expenditures for health care for 2001Q1, 2002Q1-2002Q3, 2005Q2, and 2005Q4 are excluded as well.

Table 2

Definitions of 14 Exhaustive Categories of Expenditure (abbreviations in parentheses)

Food
Alcoholic beverages (Alcbev)
Housing
Apparel
Transportation (Trans)
Health care (Health)
Entertainment (Entertn)
Personal care (Perscare)
Reading
Education (Educ)
Tobacco
Miscellaneous expenditures (Misc)
Contributions (Contrib)
Personal Insurance (Ins)

The paired t-tests in Section II that test the hypotheses that two estimated vectors of budget share coefficients are drawn from the same population are performed on a sample of 40 vector-pairs drawn at random from the population of 40!/2 different possible vector pairs. The vector-pairs forming the sample are given in Table 3.

Table 3

Sample of 40 Vector-Pairs Budget Share Coefficients 14 Categories of Expenditure 1996 Q1 - 2005 Q4

Vector-	-Pairs
1996q1-1997q2	1999q1-2004q3
1998q1-2001q3	1998q3-2003q1
2001q4-2004q4	1999q4-2002q4
1997q2-2000q2	2000q3-2005q3
2003q2-2005q3	1996q2-1997q2
2001q3-2003q3	1999q1-2004q1
2001q2-2003q1	2004q2-2004q3
2000q2-2004q1	2000q2-2003q1
2004q4-2005q4	1999q3-2002q1
1999q4-2004q1	1996q1-1996q4
1996q1-1998q4	1998q3-2005q1
2003q1-2005q4	2002q4-2004q2
1996q1-2002q4	2001q1-2004q4
2000q3-2001q3	1999q4-2001q3
2000q2-2005q2	1999q1-2005q4
1996q2-1999q2	1996q4-1999q1
1999q2-2003q1	1996q1-2004q3
1999q4-2001q2	2002q1-2003q4
1996q4-2000q2	1999q2-2002q2
2005q1-2005q3	1997q1-2003q3

70

Appendix 2

Distributions of Residuals and Differences In Expenditure and Budget Share Coefficients

As a way of including all of the coefficients for all 40 quarters in a statistical test for instability in Section II, the coefficients in the budget share regressions for the expenditure categories were "stacked" and then "split" into two vectors of 20 quarters each, 1996Q4 through 2000Q4 and 2001Q1 through 2005Q4. A paired t- test was then undertaken on differences in coefficients 20 quarters apart, 1996Q1-2001Q1, 1996Q2-2001Q2, and so on and so forth. The purpose of this appendix is to examine the distributions of these differences.

Charts showing kernel-smoothed density functions of the "split sample" differences in the estimated budget share coefficients for the 14 expenditure categories are presented in Figures 1 and 2.²⁷ Figure 1 shows the distributions for the 14 categories taken separately, while Figure 2 is for all 14 categories stacked into a single vector. The individual distributions in Figure 1 are seen to be centered either at or very close to 0, all are sharply peaked, and many have tails that are both long and asymmetrical.²⁸ Asymmetry disappears in the distribution for all 3640 differences in Figure 2, but the sharp peak and fat tails remain. Normality is obviously absent.

An alternative way of examining the differences in the "split sample" budget share coefficients is in terms of the residuals from regressing one of the vectors in the "split" on the other. The kernel-smoothed density function for this measure of "error" (i.e., for the residuals from the regression of the vector for 1996Q1-2000Q4 on the one for 2001Q1-2005Q4) is given in Figure 3. Again, normality is seen to be absent.²⁹

has been used for the smoothing parameter h. Two standard references for kernel density estimation are Silverman (1986) and Wand and Jones (1995). Ker and Goodman (2000) provide an interesting practical application to the estimation of crop insurance rates.

²⁷ The kernel-smoothed density functions have been calculated using the unit normal density function as the kernel weighting function and a "support" of 1000 intervals. Silverman's rule-of-thumb:

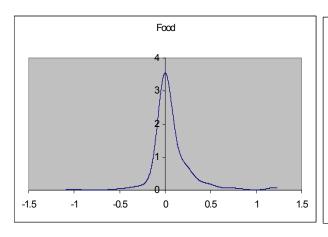
 $h = (0.9)min[std. dev., inter quartile range/1.34](N^{-1/5}),$

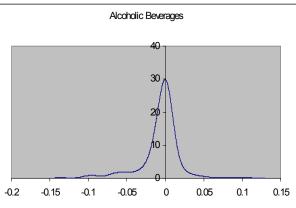
²⁸ Since differences are separated by a constant 20 quarters in time, the asymmetries reflect the trends in coefficients as depicted in the counterpart charts of Figures 1 - 14 in the text.

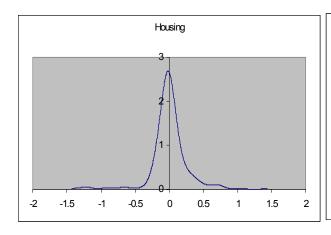
The two distributions (Figures 2 and 3) are similar in every respect, but are complementary rather than identical, as the residuals from the regression of the first vector on the second measure that part of the difference between the two vectors that is orthogonal to the second. The R² of this regression (0.6590) is identical with the R² for the regression of the vector of residuals on the vector of differences. The slope coefficients in these two regressions are 0.6590 and 1, respectively. In short, an exercise in OLS regression and linear algebra is all that is involved.

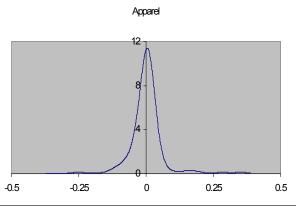
Figure 1

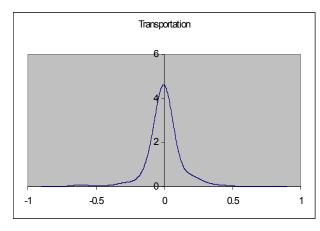
Kernel-Smoothed Distributions of Split-Sample Differences
Budget Share Coefficients
14 Categories of Expenditure











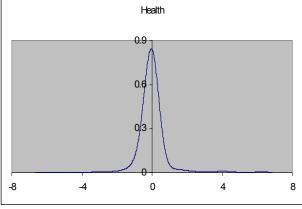
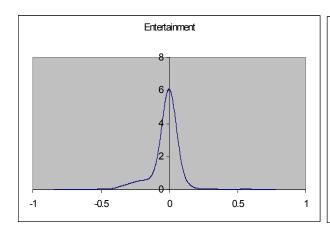
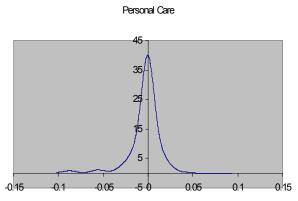
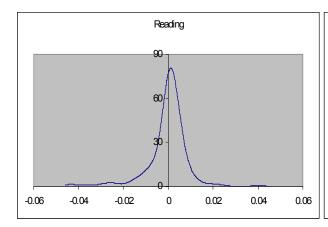
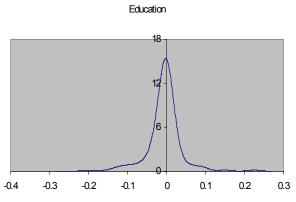


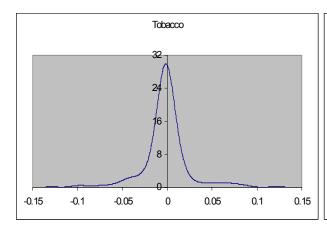
Figure 1 cont'd











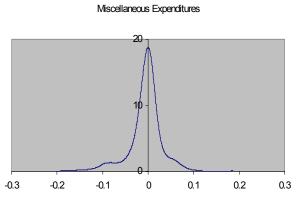
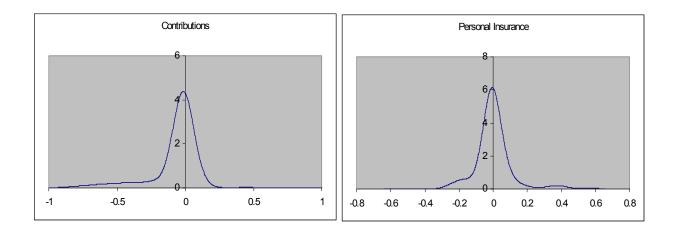
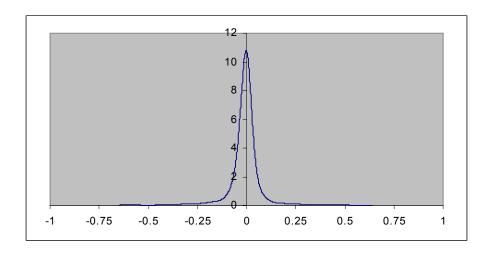


Figure 1 cont'd



Kernel-Smoothed Distribution of Stacked Split-Sample Differences
Budget Share Coefficients
14 Categories of Expenditure

Figure 2



The sharp peaks and fat tails in Figures 2 and 3 are suggestive that the "Law of Pareto" may be involved in some form, which, among other things, would suggest that the distribution triggering deviations of the ξ vectors across surveys does not possess a variance. Following Mandelbrot (1963), the standard way of testing for a Pareto element is to express the density and its support in logarithms and search for linearity. Since, in this case, the distributions in Figures 2 and 3 are symmetrical (or nearly so) about 0, this analysis can be based on a bilateral form of the Pareto distribution, whose density function has the form:³⁰

³⁰ See *CDUS*, pp. 66-70.

(1)
$$f(x) = \frac{(\alpha+2)}{2(\alpha+1)} |x|^{-(\alpha+1)} , -\infty < x < \infty, 1 < \alpha < 2,$$

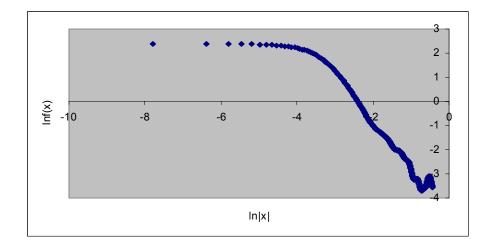
so that in logarithms:

(2)
$$\ln f(x) = A - (\alpha + 1) \ln|x|.$$

Graphs of this expression for the distribution of differences in Figure 2 are given in Figures 4 and 5, Figure 4 for x < 0 and Figure 5 for x > 0. Linearity is virtually exact for lnx between -3.5 and -1 in both "wings" of the distribution. The interesting question, of course, is the values of α in the these segments, which can be obtained by regressing lnf(x) on ln|x for -3.5 < ln|x| < -1.0. The results for the two branches (negative and positive) are given in Table 1. Since, from expression (2), the slope coefficients are equal to $-(\alpha + 1)$, estimates of α are accordingly equal to 1.52 and 1.50, respectively, values which clearly suggest that the distributions in Figures 2 and 3 do indeed not have variances.³¹

Figure 3

Kernel-Smoothed Distribution of Stacked Split-Sample Residuals
Budget Share Coefficients
14 Categories of Expenditure



As with the (single-tailed) Pareto distribution, the integral defining the variance for the bilateral version also diverges for $\alpha < 2$. See *CDUS*, p. 70. These results are actually not surprising, for distributions of residuals from Engel curves estimated with data from the surveys from 1996 through 1999 are invariably asymmetric with long tails. (See Chapter 9 of CDUS for details.) While the residuals there are obviously quite different from the "errors" under review here, it is in keeping with the conclusion in *CDUS* that assumptions regarding normality in CES survey data warrant scrutiny.

Figure 4

Double-Logarithmic Representation of Distribution in Figure 1 Negative Branch

x < 0

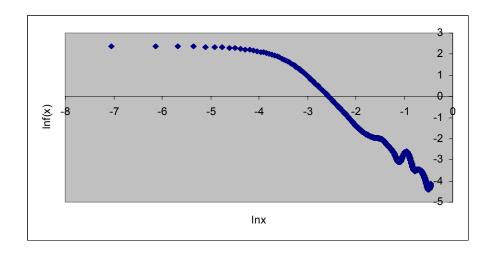
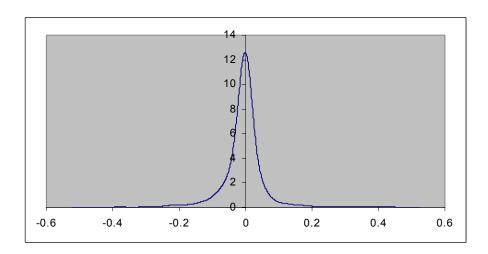


Figure 5

Double-Logarithmic Representation of Distribution in Figure 1
Positive Branch

x > 0



REFERENCES

- Mandelbrot, B. B. (1963), "New Methods in Statistical Economics," *Journal of Political Economy*, Vol. 71, pp. 421-440.
- Silverman, B. W. (1986), *Density Estimation for Statistics and Data Analysis*, Monographs on Statistics and Applied Probability 26, Chapman and Hall, London.
- Taylor, L. D. and Houthakker, H. S. (2010), Consumer Demand in The United States: Prices, Income, and Consumption Behavior, Third Edition, Springer-Verlag.
- Wand, W. P. and Jones, M. C. (1995), *Kernel Smoothing*, Monographs on Statistics and Applied Probability 60, Chapman and Hall, London.